$\hat{O} = \sum_i \lambda_i |i\rangle\langle i|$ UKM-2021

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pro $\hat{A} = 1/2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $|a+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $|a-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$\lambda_{a+} = 1/2$ $\quad$ $\lambda_{a-} = -1/2$

$$\hat{A} = \frac{1}{2}\left[\frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix}(1 \;\; 1) - \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix}(1 \;\; -1)\right]$$

$$= 1/4\left[\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right] = 1/4 \begin{pmatrix} 0 & +2 \\ 2 & 0 \end{pmatrix} = 1/2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

OK

Také platí $O_{diag} = U^{+} O \, U$ ← matice z vl. vektorů

U^{+} … transpo + c.c. ; O … originál matice

- rotace báze do báze vl. vektorů → další

- matice A, B, C jsou vyjádřeny v bázi vl. vektorů matice C, tj. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ a $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

· ~~můžeme bázi z~~

· Matice A a B ~~jsou~~ je možné získat jako

$$A = \begin{pmatrix} \langle c+|\hat{A}|c+\rangle & \langle c_+|\hat{A}|c_-\rangle \\ \langle c_-|\hat{A}|c_+\rangle & \langle c_-|\hat{A}|c_-\rangle \end{pmatrix}$$

$$\langle c+|\hat{A}|c+\rangle = \langle c+|\left[\sum_i \lambda_i |a_i\rangle\langle a_i|\right]|c_+\rangle =$$

$$= \langle c_+|\left[\tfrac{1}{2}|a+\rangle\langle a+| - \tfrac{1}{2}|a-\rangle\langle a-|\right]|c_+\rangle$$

$$= \tfrac{1}{2}\langle c_+|a_+\rangle\langle a_+|c_+\rangle - \tfrac{1}{2}\langle c_+|a_-\rangle\langle a_-|c_+\rangle$$

$$= \tfrac{1}{2}\cdot\tfrac{1}{\sqrt{2}}(1 \;\; 0)\begin{pmatrix} 1 \\ 1 \end{pmatrix}\tfrac{1}{\sqrt{2}}(1 \;\; 1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \tfrac{1}{2}\,\tfrac{1}{2}(1 \;\; 0)\begin{pmatrix} 1 \\ -1 \end{pmatrix}(1 \;\; -1)\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= 1/4 \;(1)\cdot 1 - 1/4 \cdot 1 \cdot 1 = 0$$

⋮

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$$O_{diag} = U^+ O U$$

$$\frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \rightarrow U^+ = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

$$A_{diag} = \frac{1}{4}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} =$$

$$= \frac{1}{4}\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}2 & 0\\ 0 & -2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

funguje to!

A, B, C odpovídají měření spinu podle os x, y, z, resp.
proto mají všechny vl. stavy $\pm 1/2$ [s $\hbar = 1$]

• měření do libovolné osy $\vec{n}$: $S_{\vec{n}} = \vec{n}\cdot(A, B, C)$

např. $\vec{n} = \frac{1}{\sqrt{2}}(1, 0, 1)$

$$\vec{n}\cdot\vec{v} = \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\frac{1}{\sqrt{2}}+1\\ \frac{1}{\sqrt{2}}\end{pmatrix}$$

$$S_{\vec{n}} = \frac{1}{\sqrt{2}}(A + C) = \frac{1}{\sqrt{2}}\frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$$

vl. hodnoty: $\frac{1}{2\sqrt{2}}\begin{pmatrix}1-\lambda & 1\\ 1 & -1-\lambda\end{pmatrix} \Rightarrow \begin{vmatrix}1-\lambda & 1\\ 1 & -1-\lambda\end{vmatrix} =$

$$= (1-\lambda)(-1-\lambda) - 1 = -(1-\lambda^2) -$$

vl. hodnoty $\begin{vmatrix}\frac{1}{2\sqrt{2}} - \lambda & \frac{1}{2\sqrt{2}}\\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} - \lambda\end{vmatrix} = -\left(\frac{1}{2\sqrt{2}} + \lambda\right)\left(\frac{1}{2\sqrt{2}} - \lambda\right) - \left(\frac{1}{2\sqrt{2}}\right)^2$

$$= -\frac{1}{8} + \lambda^2 - \frac{1}{8} = -\frac{1}{4} + \lambda^2 = -(\lambda + 1/2)(\lambda - 1/2)$$

$\lambda = -1/2$ $\qquad$ $\lambda = 1/2$

OK

vl. vektory: $|n+\rangle$: $\begin{pmatrix}\frac{1}{2\sqrt{2}} - 1/2 & \frac{1}{2\sqrt{2}}\\ \frac{1}{2\sqrt{2}} & -\frac{1}{2\sqrt{2}} - 1/2\end{pmatrix} \Rightarrow \frac{1}{2}\begin{pmatrix}\frac{1}{\sqrt{2}} - 1 & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} - 1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix} = 0$

$$\left(\frac{1}{\sqrt{2}} - 1\right)a + \frac{b}{\sqrt{2}} = 0 \Rightarrow b = (\sqrt{2} - 1)a$$

$\Rightarrow N\begin{pmatrix}a\\ (\sqrt{2}-1)a\end{pmatrix} \rightarrow$ normal: $N^2(a^2 + (\sqrt{2}-1)^2 a^2) = a^2(1 + 2 + 1 - 2\sqrt{2})N^2 =$

$$= a^2(4 - 2\sqrt{2})N^2 = 1$$

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• komutátory (?)

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\sigma_z \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\sigma_x \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$\sigma_y \sigma_x = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

vs. $1+2 = 2+1$ ← komutuje ↑ nekomutuje

$\sigma_x \sigma_y - \sigma_y \sigma_x = 2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 2i\,\sigma_z$

↓ značíme

$[\sigma_x, \sigma_y]$ ← komutátor $[\sigma_x, \sigma_y] = 2i\sigma_z$

↑ komutační relace

platí pro libovolný stav

$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\sigma_x \sigma_y |\uparrow\rangle = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} i \\ 0 \end{pmatrix}$

$\sigma_y \sigma_x |\uparrow\rangle = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix}$

$[\sigma_x \sigma_y - \sigma_y \sigma_x)|\uparrow\rangle = \begin{pmatrix} i \\ 0 \end{pmatrix} - \begin{pmatrix} -i \\ 0 \end{pmatrix} = 2i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ≡ stejné $2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 2i \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$|\uparrow_x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -i \\ i \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} i \\ -i \end{pmatrix} = \sigma_x \sigma_y |\uparrow_x\rangle$

$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i \\ i \end{pmatrix} = \sigma_y \sigma_x |\uparrow_x\rangle$

$[\sigma_x, \sigma_y] |\uparrow_x\rangle = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} i \\ -i \end{pmatrix} - \begin{pmatrix} -i \\ i \end{pmatrix} \right] = \frac{1}{\sqrt{2}} \{ 2 \begin{pmatrix} i \\ -i \end{pmatrix} = \sqrt{2}\, i \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$2i\sigma_z |\uparrow_x\rangle = 2i \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \sqrt{2}\, i \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ≡ OK

? užitečné, když máme složitý výraz s mnoha operátory

→ umožňují zjednodušení

Operátor na stav přímo a pomocí vl. stavů

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Mějme matici $B = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

vl. stavy: $\lambda_+ = 1/2 \Leftrightarrow |y\uparrow_y\rangle = |b_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$

$\lambda_- = -1/2 \Leftrightarrow |b_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$

test: $\frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix} -i \\ -1 \end{pmatrix} = -\frac{1}{2}\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = -\frac{1}{2}|b_-\rangle$

OK

- $B \leftrightarrow$ měření spinu podél osy y

původní stav $|\uparrow\rangle$ podél z: $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = |c_+\rangle$

$\langle b_+| B |c_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \end{pmatrix}\frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}\,2}\begin{pmatrix} i & 1 \end{pmatrix}\begin{pmatrix} 0 \\ i \end{pmatrix} =$

projekce na konečný stav; měření; původní stav

$B|c_+\rangle = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 \\ i \end{pmatrix}$ $\boxed{= \frac{1}{\sqrt{2}\,2}\, i} = \frac{1}{2}\,\frac{i}{\sqrt{2}}$

vl. číslo; koeficient

$B|c_+\rangle \neq \lambda |c_+\rangle$; $|c_+\rangle$ není vlastní stav...

ale co to je $B|c_+\rangle$

$\cancel{B|c_+\rangle = B\left[\frac{i}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] = B\left[-\frac{i}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle\right]}$

$\cancel{= \frac{1}{2}\left(-\frac{i}{\sqrt{2}}\right)|\uparrow\rangle - \frac{1}{2}\frac{1}{\sqrt{2}}|}$

$= B\begin{pmatrix} 1 \\ 0 \end{pmatrix} =$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \underset{\frac{i}{\sqrt{2}}}{c_1}\frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} + \underset{-\frac{i}{\sqrt{2}}}{c_2}\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{i}{\sqrt{2}}|b_+\rangle - \frac{i}{\sqrt{2}}|b_-\rangle$

$\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} - \frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ i \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ✓

$\Rightarrow B\begin{pmatrix} 1 \\ 0 \end{pmatrix} = B\left[\frac{i}{2}\begin{pmatrix} -i \\ 1 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} i \\ 1 \end{pmatrix}\right] = \frac{1}{2}\frac{i}{\sqrt{2}}|b_+\rangle + \frac{1}{2}\frac{i}{\sqrt{2}}|b_-\rangle$

$\rightarrow B|c_+\rangle = B\sum_i c_i |b_i\rangle = \sum_i c_i \lambda_i |b_i\rangle$

$= \frac{1}{2}\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix} + \frac{1}{2}\frac{i}{\sqrt{2}}\frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{1}{4}\begin{pmatrix} -1 \\ i \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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- Uvažujme $\langle c_+|A|c_+\rangle$, víme, kolik to vyjde, ale proč?
- ~~Va~~ Máme báze $\{|a_+\rangle, |a_-\rangle\}$ a $\{|b_+\rangle, |b_-\rangle\}$

kolik je $\sum_i |a_i\rangle\langle a_i|$ a $\sum_i |b_i\rangle\langle b_i|$ pomocí vyjádření stavů v bázi $\{|c_+\rangle, |c_-\rangle\}$?

$$|a_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \qquad |a_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$\sum_i |a_i\rangle\langle a_i| = |a_+\rangle\langle a_+| + |a_-\rangle\langle a_-| = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}(1\ 1) + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}(1\ -1)$$

$$= \frac{1}{2}\begin{pmatrix}1&1\\1&1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1&-1\\-1&1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2&0\\0&2\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$\sum_i |b_i\rangle\langle b_i| = |b_+\rangle\langle b_+| + |b_-\rangle\langle b_-|$$

$$= \frac{1}{2}\begin{pmatrix}-i\\1\end{pmatrix}(i\ \ 1) + \frac{1}{2}\begin{pmatrix}i\\1\end{pmatrix}(-i\ \ 1)$$

$$= \frac{1}{2}\begin{pmatrix}1&-i\\i&1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1&i\\-i&1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$|a_i\rangle, |b_i\rangle, |c_i\rangle$ tvoří (samostatně) úplnou bázi,

tedy $\mathbb{1} = \sum_i |a_i\rangle\langle a_i| = \sum_i |b_i\rangle\langle b_i| = \sum_i |c_i\rangle\langle c_i|$

- $\langle c_+|\hat{A}|c_+\rangle = \cancel{\sum_i |c_+} \ \langle c_+|\sum_i |a_i\rangle\langle a_i|\hat{A}\sum_j |a_j\rangle\langle a_j|c_+\rangle$

$$= \cancel{\sum_{c_+}} \sum_{ij} \langle c_+|a_i\rangle \underbrace{\langle a_i|\hat{A}|a_j\rangle}_{} \langle a_j|c_+\rangle$$

V principu součet 4 výrazů, které můžeme zapsat pomocí matice a vektorů

$\langle c_+|$ v bázi $|a_i\rangle$
$\left(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right)$

Maticová repre. operátoru $\hat{A}$ v bázi a_i (vlastních stavů)
$\begin{pmatrix}1/2&0\\0&-1/2\end{pmatrix}$ tedy diagonální

vektor c_+ vyjádřený v bázi $|a_i\rangle$
$\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}$

$$\left.\begin{aligned}\langle a_+|c_+\rangle &= \frac{1}{\sqrt{2}}(1\ 1)\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\\ \langle a_-|c_+\rangle &= \frac{1}{\sqrt{2}}(1\ -1)\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\end{aligned}\right\}$$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right)\begin{pmatrix}1/2&0\\0&-1/2\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} = \left(\frac{1}{\sqrt{2}}\ \ \frac{1}{\sqrt{2}}\right)\frac{1}{2}\begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix} = 0$$

jaště jinak

$\langle c_+ | \hat{A} | c_+ \rangle =$ vl. číslo vl. vektor

$\langle c_+ | \sum_i a_i |a_i\rangle\langle a_i| c_+\rangle = \sum_i a_i \langle c_+ | a_i \rangle \langle a_i | c_+ \rangle$

$\frac{1}{\sqrt{2}}(1 \quad 1) \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$= a_+ \langle c_+ | a_+ \rangle \langle a_+ | c_+ \rangle + a_- \langle c_+ | a_- \rangle \langle a_- | c_+ \rangle$

$= a_+ |\langle c_+ | a_+ \rangle|^2 + a_- |\langle c_+ | a_- \rangle|^2$

$(1 \quad 0)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}} \qquad (1 \quad 0)\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}$

$= a_+ \left|\frac{1}{\sqrt{2}}\right|^2 + a_- \left|\frac{1}{\sqrt{2}}\right|^2$

$= \frac{1}{2}\cdot\frac{1}{2} + \left(-\frac{1}{2}\right)\frac{1}{2} = 0$

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C v bázi $|a_i\rangle$

$U = \begin{pmatrix} a_+ & a_- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = U^+$

$C' = U^+ C U = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

~~$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$~~

$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$c'_+ = U c_+ = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$C' c'_+ = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} c'_+$ OK