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atom vodíku

13 - záření s vlnovou délkou $\lambda = 656\,nm$, jaké stavy $E_n$ & $E_m$?
$E_n$ v eV?

$$\Delta E = \hbar\omega = \hbar\, 2\pi\nu = \hbar\, 2\pi \frac{c}{\lambda}$$

$\lambda = 656\,nm$ $= 6560\,\mathring{A} = 12400 \cdot Bohr$
$c \doteq 137$

$$\Delta E[a.u.] = \frac{2\pi c}{\lambda} = \frac{2\pi \cdot 137}{12400} = \frac{861}{12400} = 0.06942\,[Ha]$$

$$\Delta E[eV] = 27.2116 \cdot E[a.u.] = 1.889\,eV$$

$E_1 = 13.605\,eV$ $\quad \frac{\Delta E}{E_1} = 0.138846$

$$E_0\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = E_0\left(\frac{m^2 - n^2}{n^2 m^2}\right)$$

$$E_0\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) = E_0\left(\frac{n^2 + 2n + 1 - n^2}{n^2(n+1)^2}\right)$$

$$E_0\left(\frac{2n+1}{n^2(n+1)^2}\right)$$

$1 \to 2 : \frac{3}{4} = 0.75$

$\boxed{2 \to 3 : \frac{5}{4 \cdot 9} = \frac{5}{36} = 0.1388}$

$3 \to 4 : \frac{7}{9 \cdot 16} = 0.0486$

$0.138\overline{8}$

14 hustota toku pravděpodobnosti pro $\psi_{nlm}$?

$$j = \frac{\hbar}{2mi}\left[\psi^* \nabla\psi - \psi \nabla\psi^*\right]$$

$$\psi_{nlm} = N_{nlm}\, R_{nl}(r)\, P_{lm}(\cos\Theta)\, e^{im\phi}$$

$$\nabla = \left(\frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \Theta}, \frac{1}{r\sin\Theta}\frac{\partial}{\partial\phi}\right)$$

pro $r$ platí $\psi^*_{nlm} = \psi_{nlm}$, $R_{nl}(r) = R^*_{nl}(r)$

$$\Rightarrow \quad \psi^* \frac{\partial}{\partial r}\psi - \psi\frac{\partial \psi^*}{\partial r} = \emptyset$$

to samé pro $\Theta$

pro $\phi$: $\quad \frac{1}{r\sin\Theta}\frac{\partial}{\partial\phi}\psi_{nlm} = \frac{1}{r\sin\Theta}\, im\, \psi_{nlm}$

$$\frac{1}{r\sin\Theta}\frac{\partial}{\partial\phi}\psi^*_{nlm} = -\frac{im}{r\sin\Theta}\psi^*_{nlm}$$

$$j_\phi = \frac{\hbar}{2m_\phi i}\left[\psi^* \frac{im}{r\sin\Theta}\psi + \psi\frac{im}{r\sin\Theta}\psi^*\right] = \frac{\hbar m}{m_{red}\, r\sin\Theta}\,\rho_{nlm}$$

15 $e$ v základním stavu, určete $\langle r \rangle$, $\langle r^2 \rangle$, $r(\rho_{max})$ T 9.1b

$$\psi_{100} = N e^{-r/a_0}$$

normování: $1 = \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \, r^2 \sin\phi \, N^2 e^{-2r/a_0} =$

$$= 4\pi N^2 \int_0^\infty r^2 e^{-2r/a_0} dr =$$

$$\int_a^b [gf]' dr = \int_a^b g'f + \int_a^b gf'$$

$$\int_a^b gf' = \int_a^b [gf]' dr - \int_a^b g'f = [gf]_a^b - \int_a^b g'f$$

$$\int_0^\infty r^2 e^{-2r/a_0} dr = \left[-\frac{a_0 r^2 e^{-2r/a_0}}{2}\right]_0^\infty - \int_0^\infty 2r\left(-\frac{a_0 e^{-2r/a_0}}{2}\right) dr$$

$$= +\int_0^\infty r a_0 e^{-2r/a_0} dr = -\left[\frac{r a_0^2 e^{-2r/a_0}}{2}\right]_0^\infty + \int_0^\infty \frac{a_0^2 e^{-2r/a_0}}{2} dr$$

$$= -\frac{a_0^3 e^{-2r/a_0}}{4}\Big|_0^\infty = \frac{a_0^3}{4}$$

$$4\pi N^2 \int_0^\infty r^2 e^{-2r/a_0} dr = 4\pi N^2 \frac{a_0^3}{4} = 1$$

$$\Rightarrow N = \sqrt{\frac{1}{\pi a_0^3}}$$

$\cancel{\langle r \rangle = \frac{1}{\pi a_0^3} \int_0^\infty r^3 e^{-2r/a_0} dr}$

$$\langle r \rangle = \frac{1}{\pi a_0^3} \int_0^\infty dr \underbrace{\int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta}_{4\pi} r^2 e^{-2r/a_0} r =$$

$$= \frac{4}{a_0^3} \int_0^\infty r^3 e^{-2r/a_0} dr = \frac{4}{a_0^3}\left[ [\,/\,] + \int_0^\infty \frac{3r^2 e^{-2r/a_0} a_0}{2} dr \right]$$

$$= \frac{4}{a_0^3} \cdot \frac{3a_0}{2} \underbrace{\int_0^\infty r^2 e^{-2r/a_0} dr}_{\frac{a_0^3}{4}} = \frac{3a_0}{2}$$

$$\langle r^2 \rangle = \frac{1}{\pi a_0^3} \int_0^\infty dr \int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta \, r^2 e^{-2r/a_0} r^2 =$$

$$= \frac{4}{a_0^3} \int_0^\infty r^4 e^{-2r/a_0} dr = \frac{4}{a_0^3}\left[ [\,/\,] + \int_0^\infty 4r^3 e^{-2r/a_0} \frac{a_0}{2} dr \right] =$$

$$= \frac{4}{a_0^3} \cdot 2a_0 \cdot \int_0^\infty r^3 e^{-2r/a_0} dr = \frac{4}{a_0^3} \cdot 2a_0 \cdot \frac{3a_0}{2} \cdot \frac{a_0^3}{4} = 3a_0^2$$

$$\rho_{max}(r) = \max \int d\theta \int d\phi \sin\theta \, r^2 |\psi_{100}|^2 = \max 4\pi r^2 |\psi_{100}|^2 =$$

$$\rightsquigarrow \max r^2 e^{-2r/a_0}$$

$$\left(r^2 e^{-2r/a_0}\right)' = 2r e^{-2r/a_0} - \frac{2r^2}{a_0} e^{-2r/a_0} = 0$$

$$1 - \frac{r}{a_0} = 0$$

$$r = a_0 \quad \text{OK}$$

17 vodík × T9. 2

$$\mu = \frac{m_e m_p}{m_e + m_p}$$

$$\hbar\omega = \frac{2\pi^2 m e^4}{h^2} = \frac{m e^4}{\hbar^2}$$

$$\hbar\omega = \hbar 2\pi \frac{c}{\lambda}$$

$$\hbar 2\pi \frac{c}{\lambda} = \frac{m e^4}{\hbar^2} \rightarrow \quad \lambda = \frac{\hbar^3 2\pi c}{m e^4}$$

$$E_1 = -\frac{1}{4\pi\varepsilon_0} \frac{\cancel{Z}^2 e^2}{2 a_B} \frac{1}{n^2} \qquad a_B = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}$$

$$E_1 = -\frac{1}{(4\pi\varepsilon_0)^2} \frac{\cancel{Z}^2 e^4 m_e}{2\hbar^2} \frac{1}{n^2} = -\frac{E_0}{n^2} \rightarrow \cancel{\lambda = \hbar 2\pi c}$$

$$= -\frac{1}{(4\pi\varepsilon_0)^2} \frac{e^4 m_e}{2\hbar^2} = \frac{(1.602 \cdot 10^{-19})^4 \cdot 9,109 \cdot 10^{-31}}{2 \cdot (1.05457\cancel{3} \cdot 10^{-34})^2} (8.9876 \cdot 10^9)^2$$

$$= \frac{1.602^4 \cdot 9,109 \cdot 8,9876^2}{2 \cdot 1,05457^2} \cdot \frac{10^{-76} \cdot 10^{-31} \cdot 10^{18}}{10^{-68}} =$$

$10^{-8} \cdot 39 + 18 \to 21$

$$= 2178,85 \cdot 10^{-21}\,J = 2,17885 \cdot 10^{-18}\,J$$

$$\frac{2,17885 \cdot 10^{-18}\,J}{1,605 \cdot 10^{-19}\,C} = 13.6\ eV \quad OK$$

$$E_1 = -\frac{1}{(4\pi\varepsilon_0)^2} \frac{\cancel{Z} e^4 \mu}{2\hbar^2}$$

$E_1 = \hbar\omega = \hbar 2\pi \frac{c}{\lambda}$ max. mění $E_0$

$$\lambda = \frac{\hbar 2\pi c}{E_1} = \frac{2\pi \cdot 1,05457 \cdot 10^{-34} \cdot 3 \cdot 10^8}{2.17885 \cdot 10^{-18}}$$

$$= \frac{2\pi \cdot 1,05457 \cdot 3}{2,17885} 10^{-8} = \cancel{3,0\ldots \cdot 10^{-8}}\,m \quad 9,123 \cdot 10^{-8}\,m$$

$$E = \hbar\omega = \hbar 2\pi \frac{c}{\lambda} = \frac{2\pi \cdot 1,05457 \cdot 10^{-34} \cdot 3 \cdot 10^8}{656 \cdot 10^{-9}} =$$

$$= 0,0303 \cdot 10^{17} = \cancel{3,0 \cdot 10^{-19}}\ 3,03 \cdot 10^{-19}\,J \quad 9.123$$

$$\cancel{9,01} \cdot 10^{-8} \left(\frac{1}{4} - \frac{1}{9}\right) = \cancel{4\ldots}$$

$1,267 \cdot 10^{-8}$

$= 12,67 \cdot 10^{-9}$

$\cancel{\lambda = \frac{\hbar 2\pi c}{E_1}}$ $E_1 \cancel{= E_0}$

T 2b

$$E_1 = \frac{\hbar 2\pi c}{\lambda_1} \qquad E_2 = \frac{\hbar 2\pi c}{\lambda_1} \cdot \frac{1}{2^2}$$

$$E_3 = \frac{\hbar 2\pi c}{\lambda_1} \cdot \frac{1}{9}$$

$$\frac{\hbar 2\pi c}{\lambda} = \frac{\hbar 2\pi c}{\lambda_1}\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_1}\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda_{23}} = \frac{1}{9{,}123\cdot 10^{-8}}\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{1}{9{,}123\cdot 10^{-8}} \cdot \frac{5}{36} = \frac{1}{9{,}128\cdot 10^{-8}} \cdot 0{,}13888$$

$$\lambda_{23} = \frac{9{,}128\cdot 10^{-8}}{0{,}1388} = 65{,}65\cdot 10^{-8} = 657\,nm \quad OK$$

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$$\mu_p = \frac{m_e m_p}{m_e + m_p} = \frac{1 \cdot 1836}{1836+1} = 0{,}999455$$

$$\mu_d = \frac{m_e \cdot m_d}{m_e + m_d} = \frac{1 \cdot 3670}{3671} = 0{,}999728$$

$$E \sim \mu \qquad \lambda_p = \lambda_0 \cdot 1{,}0005447$$

$$\lambda \sim \frac{1}{\mu} \qquad \lambda_d = \lambda_0 \cdot 1{,}0002725$$

$$\frac{\lambda_p}{\lambda_d} = 1{,}0002725$$

$$\Delta E = -\frac{E_0}{n^2} + \frac{E_0}{(m)^2} = -E_0\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

$$\Delta E = \hbar 2\pi \frac{c}{\lambda}$$

$$\frac{\hbar 2\pi c}{\lambda} = -E_0\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad E \sim \mu$$

$$\frac{1}{\lambda_p} = -E_0 k \frac{\mu_p}{\mu_0} \qquad \frac{\lambda_p}{\lambda_d} = \frac{\mu_0}{\mu_p} = 1{,}000272$$

$$\frac{1}{\lambda_d} = -E_0 k \cdot \frac{\mu_0}{\mu_0}$$