

$$\left[x \frac{d}{dx} \right]^n$$

$$n=2$$

$$\text{mit } x \frac{d}{dx} x \frac{d}{dx} \psi = x \frac{d}{dx} x \frac{d\psi}{dx} = x \left[1 \frac{d\psi}{dx} + x \frac{d^2\psi}{dx^2} \right] = \left[x \frac{d}{dx} + x^2 \frac{d^2}{dx^2} \right] \psi$$

benutze

$$\left[\frac{d}{dx}, x \right] = 1 \quad \rightarrow \quad \frac{d}{dx} x \psi - x \frac{d}{dx} \psi = 1$$

$$\frac{d}{dx} x \psi = 1 + x \frac{d}{dx} \psi$$

$$x \frac{d}{dx} x \frac{d}{dx} = x \left[1 + x \frac{d}{dx} \right] \frac{d}{dx} = x \frac{d}{dx} + x^2 \frac{d^2}{dx^2}$$

$$n=3$$

$$x \frac{d}{dx} x \frac{d}{dx} x \frac{d}{dx} \psi = x \frac{d}{dx} x \left[1 \frac{d\psi}{dx} + x \frac{d^2\psi}{dx^2} \right] = x \frac{d}{dx} \left[x \frac{d\psi}{dx} + x^2 \frac{d^2\psi}{dx^2} \right] =$$

$$= x \left[1 \frac{d\psi}{dx} + x \frac{d^2\psi}{dx^2} + 2x \frac{d^2\psi}{dx^2} + x^2 \frac{d^3\psi}{dx^3} \right] = \left[x \frac{d\psi}{dx} + 3x^2 \frac{d^2\psi}{dx^2} + x^3 \frac{d^3\psi}{dx^3} \right] \psi$$

$$x \frac{d}{dx} x \frac{d}{dx} x \frac{d}{dx} = x \left[1 + x \frac{d}{dx} \right] \left[1 + x \frac{d}{dx} \right] \frac{d}{dx} = x \frac{d}{dx} + 2x^2 \frac{d^2}{dx^2} + x^3 \frac{d^3}{dx^3} =$$

$$= x \frac{d}{dx} + 3x^2 \frac{d^2}{dx^2} + x^3 \frac{d^3}{dx^3} \quad \text{OK}$$

$$\left(\frac{d}{dx} + x \right)^2 = \left[\frac{d^2}{dx^2} + x^2 + x \frac{d}{dx} + \frac{d}{dx} x \right] = \left[\frac{d^2}{dx^2} + x^2 + 2x \frac{d}{dx} + 1 \right]$$