Operators

Find an operator that is Hermitian conjugate of the operator $\frac{d}{dx}$.

Hint: the relation between an operator and its Hermitian conjugate is $\langle\Psi|H|\Psi\rangle = \langle H^{+}\Psi|\Psi\rangle$.

Wavefunctions

Eigenfunction of p

Is the wavefunction $\psi = A\cos(kx)$ where $k \in R$ is an eigenfunction of the operator $\hat{p} = -i\hbar\frac{d}{dx}$? Is it an eigenfunction of $\hat{p}^2$? Consider a wavefunction of the form $\psi = A\cos(kx) + B\sin(kx)$. What are the conditions for A and B that will make ψ an eigenfunction of $\hat{p}$.

Eigenfunction of p, spherical coordinates

Consider a particle restricted to move on a circle. Find the eigenvalues and eigenfunctions of the operator $\hat{A} = -i\frac{d}{d\phi}$, where ϕ is the angle describing the position of the particle on the circle.

Infinite square well potential

Mean values

The ground state wavefunction of the infinitely deep square well potential on an interval $x \in (0, a)$ equals $\psi_0(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$.

- Show that this wavefunction is normalised.
- Calculate the expectation values of the position and momentum operators.

Matrix elements

Calculate the matrix elements of the dipole moment operator $q\hat{x}$, of the momentum $\hat{p}$, and of $\hat{x}^2$ for the eigenfunctions of the inifinitely deep square well potential on an interval $x \in (-a/2, a/2)$.

Hint: The eigenfunctions have the form of

$$\psi_n(x) = \sqrt{\frac{2}{a}}\cos\frac{n\pi x}{a}\,, \text{for } n = 1, 3, 5, \ldots$$

and

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}\,, \text{for } n = 2, 4, 6, \ldots$$

Matrix element A_{mn} of an operator $\hat{A}$ is obtained as $\langle m|\hat{A}|n\rangle$.