LHO — a, a^+, mat. repre., energie, stř. hodnoty, sdružená vlf a čas vývoj

ZKT1-2

$$\alpha = \sqrt{\frac{\hbar}{m\omega}}$$

$$a = \frac{1}{\sqrt{2}\alpha}\left(x + \frac{i}{m\omega}p\right) \qquad\longrightarrow\qquad x = \frac{\alpha}{\sqrt{2}}(a + a^+)$$

$$a^+ = \frac{1}{\sqrt{2}\alpha}\left(x - \frac{i}{m\omega}p\right) \qquad\qquad p = i\sqrt{\frac{m\omega\hbar}{2}}(a^+ - a)$$

$$[a, a^+] = \frac{1}{2\alpha^2}\left[\left(x + \frac{i}{m\omega}p\right)\left(x - \frac{i}{m\omega}p\right) - \left(x - \frac{i}{m\omega}p\right)\left(x + \frac{i}{m\omega}p\right)\right] =$$

x^2 a $\frac{1}{m^2\omega^2}p^2$ se požere

$$= \frac{1}{2\alpha^2}\left[x\left(-\frac{i}{m\omega}p\right) + \frac{i}{m\omega}px - x\left(\frac{i}{m\omega}p\right) + \frac{i}{m\omega}px\right] =$$

$$= \frac{1}{\alpha^2 m\omega}\left[ipx - ixp\right] = \frac{i}{\alpha^2 m\omega}\underbrace{[p, x]}_{-i\hbar} = \frac{\hbar}{\underbrace{\alpha^2}_{\frac{m\omega}{\hbar}} m\omega} = 1 \quad \text{ok}$$

$$\to [a, a^+] = 1$$

$$aa^+ = 1 + a^+a$$

$$a|n\rangle = \sqrt{n}\,|n-1\rangle$$

$$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

- maticová repre a, a^+, x, p, x^2 ?

$$\langle i|a|n\rangle = \langle i|\sqrt{n}|n-1\rangle = \sqrt{n}\,\delta_{i,n-1}$$

$$\langle i|a^+|n\rangle = \langle i|\sqrt{n+1}|n+1\rangle = \sqrt{n+1}\,\delta_{i,n+1}$$

$$a = \begin{array}{c|ccccc} i \backslash n & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 2 & 0 & 0 & 0 & \sqrt{3} & 0 \\ 3 & 0 & 0 & 0 & 0 & \sqrt{4} \\ 4 & & & 0 & & \ddots \end{array}$$

$$a^+ = \begin{array}{c|ccccc} i \backslash n & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0 & & & 0 & \\ 1 & 1 & 0 & & & \\ 2 & 0 & \sqrt{2} & 0 & & \\ 3 & 0 & 0 & \sqrt{3} & 0 & \\ & & & & & \ddots \end{array}$$

$$x = \frac{\alpha}{\sqrt{2}}(a + a^+) \qquad x = \frac{\alpha}{\sqrt{2}}\begin{bmatrix} 0 & 1 & & & \\ 1 & 0 & \sqrt{2} & & \\ & \sqrt{2} & 0 & \sqrt{3} & \\ & & \sqrt{3} & 0 & \sqrt{4} \\ & & & \sqrt{4} & 0 & \ddots \end{bmatrix}$$

$$x^2 = \frac{\alpha}{\sqrt{2}}(a + a^+)\frac{\alpha}{\sqrt{2}}(a + a^+) =$$

$$= \frac{\alpha^2}{2}\left(a^2 + aa^+ + a^+a + a^{+2}\right)$$

$$= \frac{\alpha^2}{2}\left(a^2 + a^{+2} + 2a^+a + 1\right)$$

$$x^2 = \left[\frac{\alpha}{\sqrt{2}}(a + a^+)\right]^2$$

maticově: $\frac{\alpha^2}{2}\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} & 0 & \sqrt{4} \\ & & & \sqrt{4} & \ddots \end{pmatrix}^2 = \frac{\alpha^2}{2}$
$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 1 & 0 & \sqrt{2} & 0 \\ 1 & 0 & 3 & 0 & \sqrt{6} \\ 2 & \sqrt{2} & 0 & 5 & 0 \\ 3 & 0 & \sqrt{6} & 0 & 7 \\ 4 & & & \sqrt{12} & \ddots \end{array}$$

$$\langle n|\frac{\alpha^2}{2}(a^2 + a^{+2} + 2a^+a + 1)|m\rangle =$$

$$= \frac{\alpha^2}{2}\Big[\langle n|\sqrt{m}\sqrt{m-1}|m-2\rangle + \langle n|\sqrt{m+1}\sqrt{m+2}|m+2\rangle + \langle n|2m^2|m\rangle + 1\,\delta_{nm}\Big] =$$

$$= \frac{\alpha^2}{2}\Big[\sqrt{m}\sqrt{m-1}\,\delta_{n,m-2} + \sqrt{m+1}\sqrt{m+2}\,\delta_{n,m+2} + \delta_{nm}(2m^2+1)\Big]$$

$$p^2 = -\frac{m\omega\hbar}{2}(a^+ - a)^2 = -\frac{m\omega\hbar}{2}\left[a^{+2} + a^2 - a^+a - aa^+\right] =$$

$$= -\frac{m\omega\hbar}{2}\left[a^{+2} + a^2 - 2a^+a - 1\right] = \frac{m\omega\hbar}{2}\left[2a^+a + 1 - a^{+2} - a^2\right]$$

$$H = \frac{1}{2}m\omega^2x^2 + \frac{p^2}{2m} = \frac{\omega\hbar}{4}\left[2a^+a + 1 - a^{+2} - a^2\right] + \frac{1}{2}m\omega^2\frac{\alpha^2}{2}\left[a^2 + a^{+2} + 2a^+a + 1\right] \qquad \alpha^2 = \frac{\hbar}{m\omega}$$

$$= \frac{\omega\hbar}{4}(2a^+a + 1 - a^{+2} - a^2) + \frac{\omega\hbar}{4}(2a^+a + 1 + a^2 + a^{+2}) = \frac{\omega\hbar}{2}(2a^+a + 1)$$

$\langle n|a^+a|m\rangle = \langle n|a^+\sqrt{m}|m-1\rangle = m\langle n|m\rangle = m\,\delta_{nm}$ ← op. počtu excitací slożená vlnfce LHO – střední hodnoty + čas. závislost ZKT1 – 4

$$|\psi\rangle = N\left(|0\rangle + |1\rangle + |3\rangle\right) \quad , \quad \langle x\rangle(t), \quad \langle x^2\rangle(t)$$

$$N = \frac{1}{\sqrt{3}}$$

$$\langle\psi|x|\psi\rangle \rightarrow \int_{-\infty}^{\infty}\left(e^{-x^2/2} + x\, e^{-x^2/2} + (\ \ )e^{-x^2/2}\right) x \left(\ \ \ \right) dx = \dots$$

toto samozřejmě lze dělat

$$x = \frac{\alpha}{\sqrt{2}}(a + a^+)$$

$$\langle x\rangle = \frac{1}{3}\left(\langle 0| + \langle 1| + \langle 3|\right)\frac{\alpha(a+a^+)}{\sqrt{2}}\left(|0\rangle + |1\rangle + |3\rangle\right) =$$

$$\langle 0|a|0\rangle = 0, \quad \langle 1|a|1\rangle = \sqrt{1}\langle 1|0\rangle = 0 \ \dots$$

$$= \frac{1\alpha}{3\sqrt{2}}\left[\langle 0|a|1\rangle + \langle 1|a^+|0\rangle\right] = \frac{1}{3}\frac{\alpha}{2}\left[\sqrt{1}\langle 0|0\rangle + \sqrt{1}\langle 1|1\rangle\right] =$$

$\nwarrow \pm 1$ coupling

$$= \frac{\alpha}{3}$$

$$\langle x^2\rangle = \frac{1}{3}\left[\langle 0| + \langle 1| + \langle 3|\right]\frac{\alpha^2}{2}\left(a^2 + a^{+2} + 2a^+a + 1\right)\left[|0\rangle + |1\rangle + |3\rangle\right] =$$

$$= \frac{\alpha^2}{6}\Big[\langle 0|2a^+a + 1|0\rangle + \langle 1|2a^+a + 1|1\rangle + \langle 3|2a^+a + 1|3\rangle +$$

$\nwarrow 0$ coupling

$$\langle 3|a^{+2}|1\rangle + \langle 1|a^2|3\rangle\Big]$$

$\nwarrow \pm 2$ coupling

$$= \frac{\alpha^2}{6}\left[1 + 3 + 7 + 2\sqrt{6}\right] = \frac{\alpha^2}{6}\left(11 + 2\sqrt{6}\right)$$

$$\langle 3|a^{+2}|1\rangle = \langle 3|a^+\sqrt{2}|2\rangle = \sqrt{6}$$

$$\langle 1|a^2|3\rangle = \langle 1|a\sqrt{3}|2\rangle = \sqrt{6}$$

+ čas $\quad e^{-\frac{iE_n t}{\hbar}} \to e^{-i\omega t(n+1/2)}$

$$\psi = \frac{1}{\sqrt{3}}\left(|0\rangle + |1\rangle + |3\rangle\right)$$

$$\langle x\rangle = \frac{1\alpha}{3\sqrt{2}}\left[\langle 0| + \langle 1| + \langle 3|\right](a+a^+)\left(|0\rangle + |1\rangle + |3\rangle\right) =$$

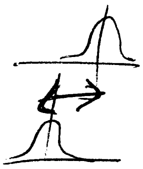
$$\langle x\rangle(t) = \frac{1}{3}\frac{\alpha}{\sqrt{2}}\left[\langle 0| e^{+\frac{i\omega t}{2}} + \langle 1| e^{+\frac{3i\omega t}{2}} + \langle 3| e^{+\frac{7i\omega t}{2}}\right]$$

$$(a+a^+)\left[|0\rangle e^{-\frac{i\omega t}{2}} + |1\rangle e^{-\frac{3i\omega t}{2}} + |3\rangle e^{-\frac{7i\omega t}{2}}\right]$$

$\langle 0|(a+a^+)|0\rangle = 0$ s časem nebo bez

$$\to \frac{1}{3}\frac{\alpha}{\sqrt{2}}\left[\langle 0|a|1\rangle e^{\frac{i\omega t}{2}} e^{-\frac{3i\omega t}{2}} + \langle 1|a^+|0\rangle e^{-\frac{i\omega t}{2}} e^{+\frac{3i\omega t}{2}}\right]$$

$$= \frac{1}{3}\frac{\alpha}{\sqrt{2}}\left[e^{-i\omega t} + e^{i\omega t}\right] = \frac{\alpha\sqrt{2}}{3}\cos(\omega t)$$



frekvence odpovídá rozdílu energie hladin