OPAKOVÁNÍ LIN. ALGEBRY

→ MATICE A JEJICH SPEKTRÁLNÍ ROZKLAD

$$\hat{A} = a_1 |1\rangle\langle 1| + a_2 |2\rangle\langle 2| + \dots$$

$$f(\hat{A}) = f(a_1) |1\rangle\langle 1| + f(a_2) |2\rangle\langle 2| + \dots$$

→ např. inverzní matice: $\hat{A}^{-1} = a_1^{-1} |1\rangle\langle 1| + a_2^{-1} |2\rangle\langle 2| + \dots$

odmocnina : $\hat{A}^{+1/2} = a_1^{1/2} |1\rangle\langle 1| + a_2^{1/2} |2\rangle\langle 2| + \dots$

exponenciela: $\exp\{\hat{A}\} = e^{a_1} |1\rangle\langle 1| + e^{a_2} |2\rangle\langle 2| + \dots$

⋮

$$\hat{A}\hat{A}^{-1} = a_1 |1\rangle\langle 1| a_1^{-1} |1\rangle\langle 1| + a_2 |2\rangle\langle 2| a_2^{-1} |2\rangle\langle 2| + \dots$$

($\langle 1|2\rangle = 0$ → křížové členy nejsou)

$$= |1\rangle\langle 1| + |2\rangle\langle 2| + \dots = \mathbb{1}$$ ...jak má být

→ obecnou hermitovskou matici můžeme napsat pomocí Pauliho σ-matic (a identity)

$$a_0 I + a_1 \sigma_x + a_2 \sigma_y + a_3 \sigma_z =$$

$$= \begin{pmatrix} a_0 + a_3 & a_1 - i a_2 \\ a_1 + i a_2 & a_0 - a_3 \end{pmatrix}$$

⇒ procvičíme si to na konkrétním příkladu:

→ např. $A = \begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}$

→ potřebujeme eigenvals eigenvec.

$$\det\begin{pmatrix} -\lambda & 1-i \\ 1+i & -\lambda \end{pmatrix} \overset{!}{=} 0$$

$$\lambda^2 - (1+i)(1-i) = 0$$

$$\lambda^2 = 2$$

$$\underline{\underline{\lambda = \pm\sqrt{2}}}$$

- $\lambda = +\sqrt{2}$

$$-\sqrt{2}\, c_1 + (1-i)\, c_2 = 0 \Rightarrow c_2 = c_1 \frac{\sqrt{2}}{1-i} = c_1 \frac{\sqrt{2}(1+i)}{2}$$

$$|c_1|^2 + |c_2|^2 = 1$$

$$|c_1|^2 + \frac{2\cdot 2}{4}|c_1|^2 = 1$$

$$\underline{\underline{|c_1| = \frac{1}{\sqrt{2}}}}$$

$$\underline{\underline{c_2 = \frac{(1+i)}{2}}}$$

- $\lambda = -\sqrt{2}$

$$+\sqrt{2}\, c_1 + (1-i)\, c_2 = 0 \Rightarrow c_2 = c_1 \frac{-\sqrt{2}(1+i)}{2}$$

$$|c_1|^2 + |c_2|^2 = 1$$

$$|c_1|^2 + |c_1|^2 = 1$$

$$\underline{\underline{|c_1| = \frac{1}{\sqrt{2}}}}$$

$$\underline{\underline{c_2 = -\frac{(1+i)}{2}}}$$

$\Rightarrow$ máme tedy:

$$\lambda_1 = +\sqrt{2} \qquad |1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1+i}{2} \end{pmatrix} \qquad \langle 1| = \left(\frac{1}{\sqrt{2}}, \frac{1-i}{2} \right)$$

$$\lambda_2 = -\sqrt{2} \qquad |2\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1+i}{2} \end{pmatrix} \qquad \langle 2| = \left(\frac{1}{\sqrt{2}}, -\frac{1-i}{2} \right)$$

=> kontrola: dostaneme matici A:

$$A \stackrel{?}{=} a_1 |1\rangle\langle 1| + a_2 |2\rangle\langle 2|$$

$$+\sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1+i}{2} \end{pmatrix} \left(\frac{1}{\sqrt{2}} \quad \frac{1-i}{2} \right) - \sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1+i}{2} \end{pmatrix} \left(\frac{1}{\sqrt{2}} \quad -\frac{1-i}{2} \right) =$$

$$= \sqrt{2} \begin{pmatrix} \frac{1}{2} & \frac{1-i}{2\sqrt{2}} \\ \frac{1+i}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} - \sqrt{2} \begin{pmatrix} \frac{1}{2} & -\frac{1-i}{2\sqrt{2}} \\ -\frac{1+i}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}}} = A \quad \checkmark$$

-> inverzní matice A^{-1}:

$$\hat{A}^{-1} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{2} & \frac{1-i}{2\sqrt{2}} \\ \frac{1+i}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} - \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{2} & -\frac{1-i}{2\sqrt{2}} \\ -\frac{1+i}{2\sqrt{2}} & \frac{1}{2} \end{pmatrix} =$$

$$= \underline{\underline{\begin{pmatrix} 0 & \frac{1-i}{2} \\ \frac{1+i}{2} & 0 \end{pmatrix}}}$$

-> kontrola $\hat{A}\hat{A}^{-1} = \mathbb{1}$:

$$\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix} \begin{pmatrix} 0 & \frac{1-i}{2} \\ \frac{1+i}{2} & 0 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}} \quad \checkmark$$

→ kde to využijeme?

$$Hc = Ec$$ → ale častěji $$Hc = ESc$$

= zobecněný vl. problém

místo toho můžeme řešit:

$$\underbrace{S^{-1}H}_{\text{není symetrické}} c = Ec$$

nebo lépe: $c = S^{-1/2}c'$

$$HS^{-1/2}c' = ES^{+1/2}c'$$

$$\boxed{\underbrace{S^{-1/2}HS^{-1/2}}_{\text{symetrické}} c' = Ec'}$$