PŘIBLIŽNÉ METODY: PORUCHOVÁ

(A) NEDEGENEROVANÉ HLADINY

$$H = H_0 + \delta H_1$$

$$E_n = E_n^{(0)} + \delta E_n^{(1)} + \delta E_n^{(2)} + \dots$$

$$|\psi_n\rangle = |\psi_n^{(0)}\rangle + \delta|\psi_n^{(1)}\rangle + \delta^2|\psi_n^{(2)}\rangle + \dots$$

přesná Schr. rce $H|\psi_n\rangle = E_n|\psi_n\rangle$

$$\Rightarrow \quad (H_0+\delta H_1)\left(|n^{(0)}\rangle + \delta|n^{(1)}\rangle + \delta^2|n^{(2)}\rangle + \dots\right) =$$

$$\left(E_n^{(0)} + \delta E_n^{(1)} + \delta^2 E_n^{(2)} + \dots\right)\left(|n^{(0)}\rangle + \delta|n^{(1)}\rangle + \delta^2|n^{(2)}\rangle + \dots\right)$$

δ^0: $H_0|n^{(0)}\rangle = E_n^{(0)}|n^{(0)}\rangle$ $\Rightarrow$ známe/umíme

δ^1: $(H_0 - E_n^{(0)})|n^{(1)}\rangle + (H_1 - E_n^{(1)})|n^{(0)}\rangle = 0$

δ^2: $(H_0 - E_n^{(0)})|n^{(2)}\rangle + (H_1 - E_n^{(1)})|n^{(1)}\rangle - E_n^{(2)}|n^{(0)}\rangle = 0$

atd.

$$\Rightarrow \quad \langle n^{(0)}|\overbrace{(H_0 - E_n^{(0)})}^{\langle n^{(0)}|E_n^{(0)}}|n^{(1)}\rangle + \langle n^{(0)}|(H_1 - E_n^{(1)})|n^{(0)}\rangle = 0$$

$$\Rightarrow \quad \boxed{E_n^{(1)} = \langle n^{(0)}|H_1|n^{(0)}\rangle}$$

$$\Rightarrow \quad \langle n^{(0)}|\overbrace{(H_0 - E_n^{(0)})}^{\langle n^{(0)}|E_n^{(0)}}|n^{(2)}\rangle + \langle n^{(0)}|(H_1 - E_n^{(1)})|n^{(1)}\rangle -$$

$$- E_n^{(2)}\langle n^{(0)}|n^{(0)}\rangle = 0$$

normalizace: $\langle\psi|\psi\rangle = 1$
nebo $\langle\psi^{(0)}|\psi\rangle = 1$ $\Big\}\Rightarrow \langle n^{(0)}|n^{(1)}\rangle = 0$

$$\Rightarrow E_n^{(2)} = \langle n^{(0)}|H_1\underbrace{|n^{(1)}\rangle}$$

ale (zatím) neznáme...

$$|n^{(1)}\rangle = \sum_i c_i^{(1)}|i^{(0)}\rangle$$ rozvineme do naší báze $\{|n^{(0)}\rangle\}$

z rce pro δ^1: pro $\langle m^{(0)}| \neq \langle n^{(0)}|$:

$$\langle m^{(0)}|(H_0 - E_n^{(0)})|n^{(1)}\rangle + \langle m^{(0)}|(H_1 - E_n^{(1)})|n^{(0)}\rangle = 0$$

$\langle m^{(0)}|E_m^{(0)}$

$$(E_m^{(0)} - E_n^{(0)})\langle m^{(0)}|n^{(1)}\rangle + \langle m^{(0)}|H_1|n^{(0)}\rangle - E_n^{(1)}\underbrace{\langle m^{(0)}|n^{(0)}\rangle}_{\delta_{mn} \Rightarrow 0} = 0$$

$$\langle m^{(0)}|\sum_i c_i^{(1)}|i^{(0)}\rangle = \sum_i c_i^{(1)}\underbrace{\langle m^{(0)}|i^{(0)}\rangle}_{\delta_{mi}} = c_m^{(1)}$$

$$(E_m^{(0)} - E_n^{(0)})c_m^{(1)} + \langle m^{(0)}|H_1|n^{(0)}\rangle = 0$$

$$\Rightarrow c_m^{(1)} = -\frac{\langle m^{(0)}|H_1|n^{(0)}\rangle}{E_m^{(0)} - E_n^{(0)}} \qquad m \neq n$$

$$\Rightarrow |n^{(1)}\rangle = \sum_i c_i|i\rangle = \sum_i -\frac{\langle i^{(0)}|H_1|n^{(0)}\rangle}{E_i^{(0)} - E_n^{(0)}}|i\rangle$$

$$\Rightarrow E_n^{(2)} = \langle n^{(0)}|H_1|n^{(1)}\rangle$$

$$\boxed{E_n^{(2)} = \sum_i \frac{\langle n^{(0)}|H_1|i^{(0)}\rangle\langle i^{(0)}|H_1|n^{(0)}\rangle}{E_n^{(0)} - E_i^{(0)}}}$$

př. LHO s poruchou:

$$H = \underbrace{\frac{p^2}{2} + \frac{x^2}{2}}_{H_0} + \underbrace{\delta x}_{H_1}$$

$$H_0|n\rangle = (n+\tfrac{1}{2})|n\rangle$$

$$\left.\begin{aligned} a &= \frac{1}{\sqrt{2}}(x+ip) \\ a^+ &= \frac{1}{\sqrt{2}}(x-ip) \end{aligned}\right\} \Rightarrow x = \frac{1}{\sqrt{2}}(a+a^+)$$

$$a|n\rangle = \sqrt{n}|n-1\rangle$$
$$a^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$E_0^{(1)} = \langle 0|x|0\rangle = \langle 0|\frac{1}{\sqrt{2}}(a+a^+)|0\rangle = 0$$

$$= \int_{-\infty}^{+\infty} \sqrt[4]{\frac{1}{\pi}}\, e^{-\frac{x^2}{2}}\, x\, \sqrt[4]{\frac{1}{\pi}}\, e^{-\frac{x^2}{2}} dx = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} e^{-x^2} x \, dx = 0$$

sudá · lichá = 0

$$E_0^{(2)} = \sum_{k\neq 0} \frac{\langle 0|x|k\rangle\langle k|x|0\rangle}{\frac{1}{2} - (k+\frac{1}{2})} = \sum_{k\neq 0} \frac{|\langle 0|x|k\rangle|^2}{-k} =$$

$$\langle 0|x|k\rangle = \frac{1}{\sqrt{2}}\langle 0|(a+a^+)|k\rangle =$$
$$= \frac{1}{\sqrt{2}}\left(\sqrt{k}\langle 0|k-1\rangle + \sqrt{k+1}\langle 0|k+1\rangle\right)$$

$\rightarrow$ přispěje pouze $k=1$

$$= \frac{|\frac{1}{\sqrt{2}}|^2}{-1} = -\frac{1}{2}$$

$\Rightarrow$ energie $E_0 \approx E_0^{(0)} + \delta E_0^{(1)} + \delta^2 E_0^{(2)}$

$$\frac{1}{2} + \delta \cdot 0 + \delta^2 \cdot (-\tfrac{1}{2}) = \frac{1}{2} - \frac{1}{2}\delta^2$$

$\rightarrow$ pro $\delta = 0{,}1$: $E_0 \approx 0{,}495$

$\delta = 0{,}01: \; \underline{E_0 \simeq 0{,}49995}$

(B) DEGENEROVANÉ HLADINY

více hladin se stejnou energií E_n $|n\rangle \to \sum d_i |n_i\rangle$

$\Rightarrow$ řešíme vl. problém

$$\begin{pmatrix} W_{11} - E_n^{(1)} & W_{12} & \cdots & \\ W_{21} & W_{22} - E_n^{(1)} & & \\ W_{31} & W_{32} \cdots & \ddots & \\ & & & W_{NN} E_n \end{pmatrix} \begin{pmatrix} d_1 \\ \vdots \\ d_N \end{pmatrix} = 0$$

↑ příspěvky jednotlivých stavů

př. spřažené oscilátory:

$$H = \underbrace{\frac{p_x^2}{2} + \frac{x^2}{2} + \frac{p_y^2}{2} + \frac{y^2}{2}}_{H_0} + \delta \underbrace{x^2 y^2}_{H_1}$$

vl. stavy $|i\rangle_x \otimes |j\rangle_y \equiv |ij\rangle$

$$H_0 |ij\rangle = (H_x |i\rangle) |j\rangle + |i\rangle (H_y |j\rangle) =$$

$$= (i + \tfrac{1}{2}) |ij\rangle + (j + \tfrac{1}{2}) |ij\rangle$$

$$= \underbrace{(i + j + 1)}_{E_{ij} \equiv E_{i+j}} |ij\rangle$$

$\Rightarrow E_0 = 1 \quad |00\rangle$ nedeg.

$E_1 = 2 \quad |10\rangle, |01\rangle$ deg.

$E_2 = 3 \quad |20\rangle, |11\rangle, |02\rangle$ deg.

$\vdots$

→ pro první exc. stav:

$$\begin{pmatrix} \langle 10|H_1|10\rangle - E_1^{(1)} & \langle 10|H_1|01\rangle \\ \langle 01|H_1|10\rangle & \langle 01|H_1|01\rangle - E_1^{(1)} \end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = 0$$

$$\langle 10|H_1|10\rangle = \langle 10|x^2y^2|10\rangle =$$

$$= \langle 1|x^2|1\rangle\langle 0|y^2|0\rangle =$$

$$= \langle 1|_x \tfrac{1}{2}(aa + a^\dagger a + aa^\dagger + a^\dagger a^\dagger)|1\rangle_x \times$$

$$\times \langle 0|\tfrac{1}{2}(bb + b^\dagger b + bb^\dagger + b^\dagger b^\dagger)|0\rangle =$$

$$= \frac{1}{4}\underbrace{\langle 1|(a^\dagger a + aa^\dagger)|1\rangle}_{1+2}\underbrace{\langle 0|bb^\dagger|0\rangle}_{=1} =$$

$$= \frac{3}{4}$$

$$\langle 01|H_1|01\rangle = \dots = \frac{3}{4}$$

$$\langle 01|H_1|10\rangle = (\langle 10|H_1|01\rangle)^* = 0$$

$$\Rightarrow \begin{pmatrix} \frac{3}{4} - E_1^{(1)} & 0 \\ 0 & \frac{3}{4} - E_1^{(1)} \end{pmatrix}\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix} = 0$$

$$\Rightarrow \det(\quad) \overset{!}{=} 0$$

$$\left(\frac{3}{4} - E_1^{(1)}\right)^2 = 0$$

$E_1^{(1)} = \frac{3}{4}$ → nedojde k sejmutí degenerace, hladiny se jen posunou

→ $E_1 \approx E_1^{(0)} + \delta E_1^{(1)} = 2 + \delta\frac{3}{4}$

pro $\delta = 0{,}1$: $2{,}075$

(pr.) 3D LHO v magn. polji

$$H = H_1 + H_2 = \frac{1}{2}(p_x^2 + x^2) + \frac{1}{2}(p_y^2 + y^2) + \frac{1}{2}(p_z^2 + z^2)$$

$$a = \frac{1}{\sqrt{2}}(x + ip)$$

$$a^+ = \frac{1}{\sqrt{2}}(x - ip)$$

$$[a, a^+] = 1$$

$$H = a_x^+ a_x + \frac{1}{2} + a_y^+ a_y + \frac{1}{2} + a_z^+ a_z + \frac{1}{2}$$

$$H = a_x^+ a_x + a_y^+ a_y + a_z^+ a_z + \frac{3}{2}$$

$$L_i = \varepsilon_{ijk} x_j p_k$$

$$L_x = y p_z - z p_y$$

$$y = \frac{1}{\sqrt{2}}(a_y + a_y^+) \qquad p_y = \frac{i}{\sqrt{2}}(a_y^+ - a_y)$$

$$z = \frac{1}{\sqrt{2}}(a_z + a_z^+) \qquad p_z = \frac{i}{\sqrt{2}}(a_z^+ - a_z)$$

$$\underline{\underline{L_x}} = \frac{i}{2}(a_y + a_y^+)(a_z^+ - a_z) -$$

$$- \frac{i}{2}(a_z + a_z^+)(a_y^+ - a_y) =$$

$$= \frac{i}{2}\Big[\underline{a_y a_z^+} + \cancel{a_y^+ a_z^+} - \cancel{a_y a_z} - \underline{a_y^+ a_z} -$$

$$\ldots a_z a_y - a_z \cancel{a_y^\dagger} + \cancel{a_z} a_y + a_z^\dagger a_y) =$$

$$= i\,(a_y a_z^\dagger - a_y^\dagger a_z)$$

energie nepor. 3D LHO:

$$E_n = n_x + n_y + n_z + \frac{3}{2}$$

$\Rightarrow$ první excitovaný stav:

$$E_1 = \frac{5}{2} \quad \{|100\rangle, |010\rangle, |001\rangle\}$$

PM pro degenerované hladiny

$$\langle n_x' n_y' n_z' | a_y a_z^\dagger | n_x n_y n_z \rangle =$$

$$= \underbrace{\langle n_x' | n_x \rangle}_{\delta_{n_x' n_x}} \underbrace{\langle n_y' | n_y - 1 \rangle}_{\delta_{n_y' n_y - 1}} \underbrace{\langle n_z' | n_z + 1 \rangle}_{\delta_{n_z' n_z + 1}} \sqrt{n_y (n_z + 1)}$$

$$\langle n_x' n_y' n_z | a_z a_y^\dagger | n_x n_y n_z \rangle =$$

$$= \langle n_x' | n_x \rangle \langle n_y' | n_y + 1 \rangle \langle n_z' | n_z - 1 \rangle \sqrt{(n_y + 1) n_z}$$

$\Rightarrow$ dostaneme matici

$$\begin{array}{c|ccc} & |100\rangle & |010\rangle & |001\rangle \\ \hline \langle 100| & 0 & 0 & 0 \\ \langle 010| & 0 & 0 & 1 \end{array}$$

$\langle 001| \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}$

$$\Rightarrow \det \begin{vmatrix} -\lambda & 0 & 0 \\ 0 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = -\lambda \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} =$$

$$= -\lambda \cdot (\lambda^2 - 1) \stackrel{!}{=} 0$$

$$\Rightarrow \lambda = 0, \pm 1$$

$$\Rightarrow \underline{\underline{E_1^{(1)} = 0, \pm \frac{qB}{2m}}}$$