# ZEEMANŮV JEV

⇒ štěpení spektrálních čar vlivem vnějšího magn. pole
⇒ sejmeme degeneraci v m

→ minulá cvičení: sodík → FS, HFS
↳ budeme se zabývat 3S, 3P stavy a přechody mezi nimi

statické magnetické pole $\vec{B} = B\vec{z}$ $(B>0)$

⇒ Zeemanův hamiltonián:

$$H_Z = H_Z(e) + H_Z(j) =$$

$$\boxed{H_Z = \mu_B B\,(L_z + 2S_z) - g_I \mu_n B I_z}$$

$$\mu_B = \frac{q}{2m} \qquad \mu_n = \frac{m}{M}\mu_B$$

→ pro popis systému: $\underbrace{\underbrace{\vec{L}, \vec{S}}_{\vec{J}}, \vec{I}}_{\vec{F}}$


a hamiltonián obecně: $H_0 + \underbrace{H_{FS}}_{\vec{L}\cdot\vec{S}} + \underbrace{H_{HFS}}_{(\vec{L},\vec{S})\cdot\vec{I}} + \underbrace{H_Z}_{L_z,\, S_z,\, I_z}$

⇒ jakou bázi zvolíme?

• $\{ |I, m_I, L, m_L, S, m_S\rangle \}$

• $\{ |\underbrace{I, m_I}_{\text{jádro}}, \underbrace{J, m_J}_{e^-}\rangle \}$

• $\{ |I, J, F, M_F\rangle \}$ ← složíme $\vec{F} = \vec{I} + \vec{J}$

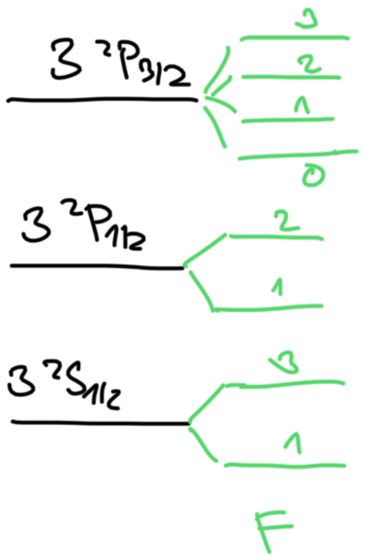
## ① SLABÉ POLE (lineární Zeemanův jev)

$$H_0 > H_{FS} > H_{HFS} \gg H_Z$$

$\Rightarrow$ $H_0 + H_{FS} + H_{HFS}$ = neporušený ham.

$H_Z$ = porucha

$\Rightarrow$ vezmeme bázi $\{ |I, J, F, M_F\rangle \}$




změna (posun/rozštěpení) stavu $|I, J, F, M_F\rangle$:

$$\varepsilon_1 = \langle I, J, F, M_F | H_Z | I, J, F, M_F\rangle$$

$$= \langle I, J, F, M_F | \mu_B B (L_z + 2S_z) | I, J, F, M_F\rangle +$$

$$+ \langle I, J, F, M_F | (-g_I \mu_n B I_z) | I, J, F, M_F\rangle$$

$\Rightarrow$ potřebujeme spočítat:

$$\langle I, J, F, M_F | L_z | I, J, F, M_F \rangle$$

trik: $\langle L_z \rangle = \frac{\langle \vec{L}\cdot\vec{J}\rangle}{J(J+1)} \langle J_z \rangle$ projection theorem

a analogicky $\langle S_z \rangle = \frac{\langle \vec{S}\cdot\vec{J}\rangle}{J(J+1)} \langle J_z \rangle$

ale v bázi $|I, J, F, M_F\rangle$ máme $F_z$ (ane $J_z, I_z$)

$$\Rightarrow \langle L_z \rangle = \frac{\langle \vec{L}\cdot\vec{J}\rangle}{J(J+1)} \cdot \frac{\langle \vec{J}\cdot\vec{F}\rangle}{F(F+1)} \langle F_z \rangle$$

$$\langle S_z \rangle = \frac{\langle \vec{S}\cdot\vec{J}\rangle}{J(J+1)} \cdot \frac{\langle \vec{J}\cdot\vec{F}\rangle}{F(F+1)} \langle F_z \rangle$$

$$\langle I_z \rangle = \frac{\langle \vec{I}\cdot\vec{F}\rangle}{F(F+1)} \langle F_z \rangle$$

$$\Rightarrow \langle \vec{J}\cdot\vec{F}\rangle, \langle \vec{L}\cdot\vec{J}\rangle, \langle \vec{S}\cdot\vec{J}\rangle, \langle \vec{I}\cdot\vec{F}\rangle = ?$$

$$\langle \vec{L}\cdot\vec{J}\rangle = \langle \vec{L}\cdot(\vec{L}+\vec{S})\rangle = \langle L^2 + \vec{L}\cdot\vec{S}\rangle =$$

$$= \langle L^2 + \tfrac{1}{2}(J^2 - L^2 - S^2)\rangle =$$

$$= \tfrac{1}{2}\langle J^2 + L^2 - S^2\rangle = \tfrac{1}{2}[j(j+1) + l(l+1) - s(s+1)]$$

$$\langle \vec{S}\cdot\vec{J}\rangle = \langle \vec{S}\cdot(\vec{L}+\vec{S})\rangle = \langle S^2 + \vec{L}\cdot\vec{S}\rangle =$$

$$= \langle S^2 + \tfrac{1}{2}(J^2 - L^2 - S^2)\rangle =$$

$$= \tfrac{1}{2}\langle J^2 + S^2 - L^2\rangle = \tfrac{1}{2}[j(j+1) + s(s+1) - l(l+1)]$$

$$\langle \vec{J}\cdot\vec{F}\rangle = \dots = \tfrac{1}{2}\langle F^2 + J^2 - I^2\rangle = \tfrac{1}{2}[f(f+1) + j(j+1) - i(i+1)]$$

$$\langle \vec{I}\cdot\vec{F}\rangle = \dots = \tfrac{1}{2}\langle F^2 + I^2 - J^2\rangle = \tfrac{1}{2}[f(f+1) + i(i+1) - j(j+1)]$$

$$\varepsilon_1^{(e)} = \langle I, J, F, M_F | H_Z^{(e)} | I, J, F, M_F \rangle =$$

$$= \mu B \langle F, J, F, M_F | (L_z + 2S_z) | I, J, F, M_F \rangle =$$

$$= \mu_B B \underbrace{\frac{\{J(J+1) + L(L+1) - S(S+1)\} + 2\{J(J+1) + S(S+1) - L(L+1)\}}{2J(J+1)}}_{\frac{3}{2} + \frac{S(S+1) - L(L+1)}{2J(J+1)} \equiv g_J \text{ Landeův faktor (pro jemnou str.)}} \langle J_z \rangle$$

$$= \mu_B B g_J \langle J_z \rangle =$$

$$= \mu_B B g_J \underbrace{\frac{F(F+1) + J(J+1) - I(I+1)}{2F(F+1)}}_{\to \alpha} M_F$$

$$\varepsilon_1^{(n)} = \langle I, J, F, M_F | H_Z^{(n)} | I, J, F, M_F \rangle =$$

$$= -\mu_n B \langle I_z \rangle =$$

$$= -\mu_n B \underbrace{\frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)}}_{\to \beta} M_F$$

$$\Rightarrow \varepsilon_1 = \varepsilon_1^{(e)} + \varepsilon_1^{(n)} =$$

lineární!

$$= \mu_B B \underbrace{\left[ g_J \alpha - \frac{\mu_n}{\mu_B} g_I \beta \right]}_{\equiv g_F \text{ Landeův faktor pro hyperjemnou str.}} M_F = \mu_B B g_F M_F$$

dále zanedbáme člen pro jádro a spočteme $g_J, g_F$ pro $3\,^2S_{1/2}$, $3\,^2P_{1/2}$, $3\,^2P_{3/2}$:

$3\,^2S_{1/2}$: $L=0$ $S=\frac{1}{2}$ $J=\frac{1}{2}$ $I=\frac{3}{2}$ $F=1,2$

$F=1$: $g_J = 2$ $g_F = -\frac{1}{2}$

$F=2$: $g_J = 2$ $g_F = +\frac{1}{2}$

$3\,^2P_{1/2}$: $L=1$ $s=\frac{1}{2}$ $J=\frac{1}{2}$ $I=\frac{3}{2}$ $F=1,2$

$F=1$: $g_J=\frac{2}{3}$ $g_F=-\frac{1}{6}$

$F=2$: $g_J=\frac{2}{3}$ $g_F=+\frac{1}{6}$

$3\,^2P_{3/2}$: $L=1$ $s=\frac{1}{2}$ $J=\frac{3}{2}$ $I=\frac{3}{2}$ $F=0,1,2,3$

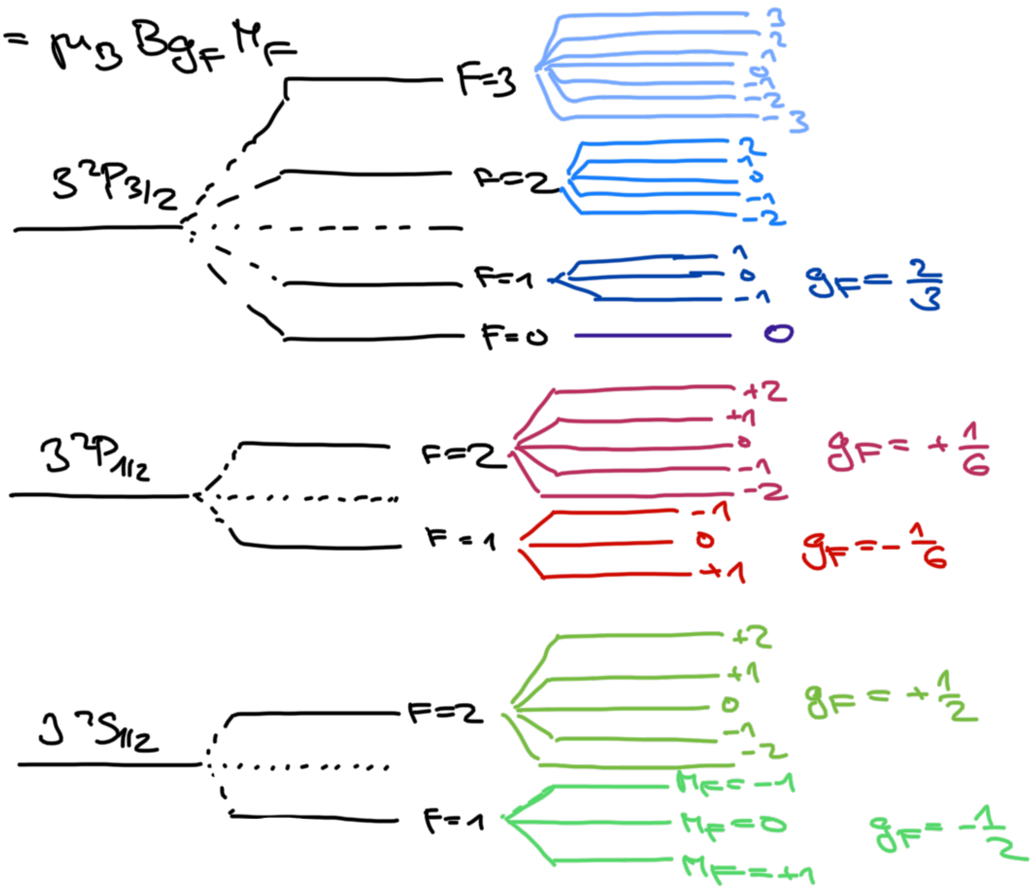
$g_J=\frac{4}{3}$

$g_F$ nezávisí na $F$ pro $I=J$:

$$g_F=\frac{F(F+1)+J(J+1)-I(I+1)}{2F(F+1)}\,g_J=$$

$$=\frac{1}{2}g_J=\frac{2}{3}$$

$E_1=\mu_B B g_F M_F$




$\Rightarrow$ úplné sejmutí degenerace

## ② SILNÉ POLE (Back-Goudsmit)

$$\underbrace{H_0 > H_{FS}}_{\text{neporušený ham.}} > \underbrace{H_Z(e^-)}_{\text{1. porucha}} \gg \underbrace{H_{HFS} > H_Z(n)}_{\text{2. porucha}}$$

$$H_Z(e^-) = \mu_B B\,(L_z + 2S_z) = \mu_B B g_J \hat{J}_z$$

$$H_{HFS} + H_Z(n) = A\,\vec{I}\cdot\vec{J} - \mu_n B g_I I_z$$

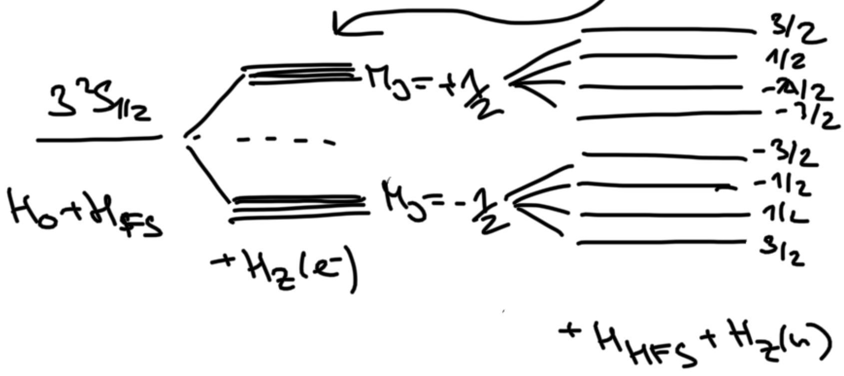
$\Rightarrow$ báze $\{ |I, M_I, J, M_J\rangle \}$

porucha #1: $H_Z(e^-)$

$$\Rightarrow \varepsilon_1 = \langle I, M_I, J, M_J | \mu_B B g_J \hat{J}_z | I, M_I, J, M_J \rangle =$$

$$= \mu_B B g_J M_J$$

$\Rightarrow$ degenerace v $M_J$ se zruší, ale stále deg. v $M_I$




porucha #2: $H_Z(n) + H_{HFS}$

proč ne báze $\{ |F, M_F\rangle \}$? $\rightarrow$ míchá stavy s různým $M_F$

$$M_F = M_J + M_I$$

$\Rightarrow$ př.: $M_J = 1/2$, $M_I = 1/2$ $\Rightarrow$ $M_F = 1$

$\Rightarrow M_J = \frac{1}{2} \quad M_I = \frac{1}{2} \Rightarrow M_F = 1$

$M_J = -1/2 \quad M_I = \frac{3}{2} \Rightarrow M_F = 1$

$\Rightarrow$ použijeme $\{|I, M_I, J, M_J\rangle\}$

$$H_2 = A\,\vec{J}\cdot\vec{I} - \mu_B B g_I I_z$$

$$\mathcal{E}_2 = \langle I, M_I, J, M_J | A \underbrace{\vec{J}\cdot\vec{I}}_{} - \mu_B B g_I I_z | I, M_I, J, M_J\rangle$$

$\rightarrow M_I$

$$\frac{1}{2}(I_+J_- + I_-J_+) + I_zJ_z$$

$$\langle I, M_I, J, M_J | \frac{1}{2}\underbrace{(I_+J_- + I_-J_+)}_{\downarrow} + I_zJ_z | I, M_I, J, M_J\rangle =$$

$M_I \cdot M_J$

$\Rightarrow$ dá $= 0$

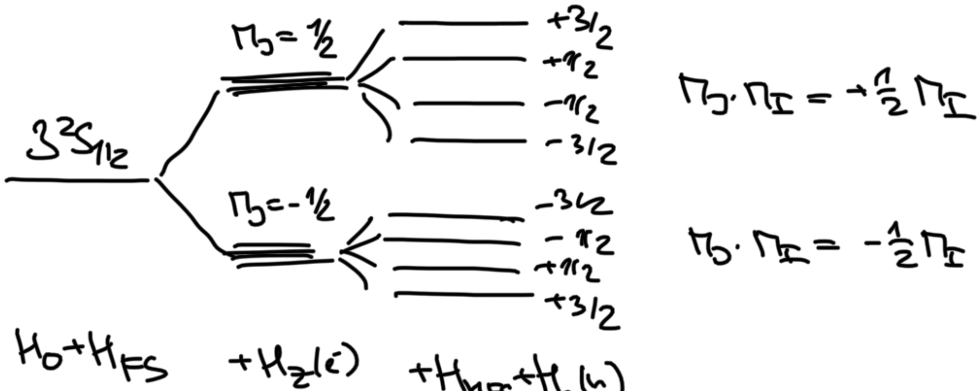
protože se pohybujeme v podprostoru $M_J =$ konst.

$$\Rightarrow \mathcal{E}_2 = A M_I M_J - \mu_B B g_I M_I$$

$\Rightarrow$ úplné sejmutí degenerace

$\Rightarrow$ celkově:

$$\Delta E = \mu_B B \left(g_J M_J - \frac{\mu_N}{\mu_B} g_I M_I\right) + A M_J M_I$$




(?) STŘEDNÍ SILNÁ POLE

## ③ STŘEDNĚ SILNÁ POLE

$$H_0 > H_{FS} > H_Z \approx H_{HFS}$$

$H_0 + H_{FS}$ neporušený ham.

$$H_1 = H_Z + H_{HFS} = \mu_B B g_J J_z + A\vec{J}\cdot\vec{I} - \mu_N g_I B I_z$$

pro $J = 1/2$:

$m_J = \pm 1/2 \quad m_I = \pm 3/2, \pm 1/2 \Rightarrow$ celkem 8 stavů

$|I, m_I, J, m_J\rangle$

$\Rightarrow$ matice 8×8 $\quad \langle m_I, m_J | H_1 | m_I\, m_J\rangle$

$$H_1 = \mu_B B\Big(\underset{\text{diag}}{g_J J_z} - \underset{\text{diag}}{\frac{\mu_N}{\mu_B} g_I I_z}\Big) + A\Big(\underset{\text{diag}}{I_z J_z} + \frac{1}{2}\underbrace{(I_+ J_- + I_- J_+)}_{\text{mimodiag.}}\Big)$$

ale mísí jen $m_I \pm 1, m_J \pm 1$ !
a $m_F = m_I + m_J$ se zachovává !

$\Rightarrow$ matice bude mít tvar:

$$\begin{array}{c|cccccccc}
m_J = & 1/2 & -1/2 & 1/2 & -1/2 & 1/2 & -1/2 & 1/2 & -1/2 \\
m_I = & 3/2 & 3/2 & 1/2 & 1/2 & -1/2 & -1/2 & -3/2 & -3/2 \\
m_F & 2 & 1 & 1 & 0 & 0 & -1 & -1 & -2 \\
\hline
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\end{array}$$

$\Rightarrow$ 2 vl. čísla a vl. stavy jasné:

$M_F = +2 \quad |3/2, 1/2\rangle$
$M_F = -2 \quad |-3/2, -1/2\rangle$

$\Delta E = M_I M_J A + \mu_B B g_F M_F = \frac{3}{4} A + \mu_B B g_F (\pm 2)$

$\Rightarrow$ lineární v B a vl. vektory nezávislé na B

$\Rightarrow$ 3x2 stavy lin. k. vždy dvou stavů
s $M_F$ = konst.