SKLÁDÁNÍ MOMENTŮ HYBNOSTI

SKLÁDÁNÍ DVOU SPINŮ 1/2

skládání např. spin e^- $s_e = 1/2$ + spin jádra $s_j = 1/2$

2 možné stavy $\{|\uparrow\rangle, |\downarrow\rangle\}$ ⊗ 2 možné stavy $\{|\uparrow\rangle, |\downarrow\rangle\}$

⇒ dohromady $2 \times 2 = 4$ možné stavy

$$\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$$

⇒ ... ⇒ při výpočtu HFS jsme získali LK

$$\lambda = \frac{1}{4} \quad |\uparrow\uparrow\rangle, \quad \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right), \quad |\downarrow\downarrow\rangle \qquad \lambda = -\frac{3}{4} \quad \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

(*)

⇒ zavedeme operátor celk. spinu $\vec{S} = \vec{S}_e + \vec{S}_j$

⇒ $S_z = S_{e,z} + S_{j,z}$

⇒ $S^2 = (\vec{S}_e + \vec{S}_j)^2 = S_e^2 + S_j^2 + 2\vec{S}_e \cdot \vec{S}_j$

$$\vec{S}_e \cdot \vec{S}_j = \frac{1}{2}\left(S_+^e S_-^j + S_-^e S_+^j\right) + S_z^e S_z^j$$

$$S^2 = S_e^2 + S_j^2 + \left(S_{e,+} S_{j,-} + S_{e,-} S_{j,+}\right) + 2 S_{z,e} S_{j,e}$$

⇒ stavy (*) popíšeme pomocí S, M

$$S^2 |S, M\rangle = S(S+1) |S, M\rangle$$
$$S_z |S, M\rangle = M |S, M\rangle$$

$$\underline{\underline{S_z |\uparrow\uparrow\rangle}} = (S_{e,z} + S_{j,z}) |\uparrow\uparrow\rangle =$$

$$= (S_{e,z}|\uparrow\rangle) \otimes |\uparrow\rangle + |\uparrow\rangle \otimes (S_{j,z}|\uparrow\rangle) =$$

$$= \frac{1}{2} |\uparrow\uparrow\rangle + \frac{1}{2} |\uparrow\uparrow\rangle =$$

$$= \underline{\underline{1 \, |\uparrow\uparrow\rangle}}$$

$$\underline{\underline{S_z |\downarrow\downarrow\rangle}} = (S_{e,z}|\downarrow\rangle) \otimes |\downarrow\rangle + |\downarrow\rangle \otimes (S_{j,z}|\downarrow\rangle) =$$

$$= -\frac{1}{2} |\downarrow\downarrow\rangle - \frac{1}{2} |\downarrow\downarrow\rangle =$$

$$= \underline{\underline{-1 \, |\downarrow\downarrow\rangle}}$$

$$\underline{\underline{S_z \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)}} = \frac{1}{\sqrt{2}} \Big[S_{e,z}|\uparrow\rangle \otimes |\downarrow\rangle + |\uparrow\rangle \otimes S_{j,z}|\downarrow\rangle$$

$$- (S_{e,z}|\downarrow\rangle \otimes |\uparrow\rangle + |\downarrow\rangle \otimes S_{j,z}|\uparrow\rangle) \Big] =$$

$$= \frac{1}{\sqrt{2}} \Big[\frac{1}{2} |\uparrow\downarrow\rangle - \frac{1}{2} |\uparrow\downarrow\rangle - \big(-\frac{1}{2} |\downarrow\uparrow\rangle + \frac{1}{2} |\downarrow\uparrow\rangle\big) \Big] =$$

$$= \underline{\underline{0}}$$

$$\underline{S_z \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)} = \dots = \underline{0}$$

$$S^2 = S_e^2 + S_j^2 + (S_{e,+} S_{j,-} + S_{e,-} S_{j,+}) + 2 S_{e,z} S_{j,z}$$

$$\underline{\underline{S^2 |\uparrow\uparrow\rangle}} = S_e^2 |\uparrow\rangle \otimes |\uparrow\rangle + |\uparrow\rangle \otimes S_j^2 |\uparrow\rangle +$$

$$+ S_{e,+}|\uparrow\rangle \otimes S_{j,-}|\uparrow\rangle + S_{e,-}|\uparrow\rangle \otimes S_{j,+}|\uparrow\rangle +$$

$$+ 2 S_{e,z}|\uparrow\rangle \otimes S_{j,z}|\uparrow\rangle =$$

$$= \frac{1}{2}\left(\frac{1}{2}+1\right) |\uparrow\uparrow\rangle + \frac{1}{2}\left(\frac{1}{2}+1\right) |\uparrow\uparrow\rangle +$$

$$+ 0 + 0 + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} |\uparrow\uparrow\rangle =$$

$$= 2|\uparrow\uparrow\rangle = \underline{\underline{1(1+1)|\uparrow\uparrow\rangle}}$$

$$\underline{\underline{S^2|\downarrow\downarrow\rangle}} = \dots = 2|\downarrow\downarrow\rangle = \underline{\underline{1(1+1)|\downarrow\downarrow\rangle}}$$

$$S^2|\uparrow\downarrow\rangle = S_e^2|\uparrow\rangle \otimes |\downarrow\rangle + |\uparrow\rangle \otimes S_j^2|\downarrow\rangle +$$
$$+ S_{e,+}|\uparrow\rangle \otimes S_{j,-}|\downarrow\rangle + S_{e,-}|\uparrow\rangle \otimes S_{j,+}|\downarrow\rangle +$$
$$+ 2 S_{e,z}|\uparrow\rangle \otimes S_{j,z}|\downarrow\rangle =$$
$$= \tfrac{3}{4}|\uparrow\downarrow\rangle + \tfrac{3}{4}|\uparrow\downarrow\rangle + 0 + |\downarrow\uparrow\rangle - \tfrac{1}{2}|\uparrow\downarrow\rangle =$$
$$= |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

$$S^2|\downarrow\uparrow\rangle = \dots = |\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$$

$$\Rightarrow S^2 \tfrac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) = \underline{\underline{1(1+1)\tfrac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)}}$$
$$S^2 \tfrac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) = \underline{\underline{0}}$$

⇒ stavy můžeme označit pomocí $s = 0, 1$
$M = -1, 0, 1$

$$\left.\begin{aligned} |1,+1\rangle &= |\uparrow\uparrow\rangle \\ |1,0\rangle &= \tfrac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \\ |1,-1\rangle &= |\downarrow\downarrow\rangle \end{aligned}\right\} \text{triplet}$$

$$|0,0\rangle = \tfrac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \ \} \text{ singlet}$$

$$\boxed{\text{SKLÁDÁNÍ M.H. } j_1 = 1/2 \text{ a } j_2 = 1/2 \text{ PODLE NÁVODU V KNIZE DOC. ZAMASTILA}}$$

celkový moment hybnosti $\vec{J} = \vec{J}_1 + \vec{J}_2$

⇒ ÚMKO $\{J^2, J_z, J_1^2, J_2^2\}$

Hledáme společné vlastní stavy $|\psi\rangle = |j, m (j_1, j_2)\rangle$ takové, že:

$$J^2 |j, m (j_1 j_2)\rangle = j(j+1) |j m (j_1 j_2)\rangle$$
$$J_z |j m, (j_1 j_2)\rangle = m |j m, (j_1 j_2)\rangle$$
$$J_1^2 |j m (j_1 j_2)\rangle = j_1(j_1+1) |j m (j_1 j_2)\rangle$$
$$J_2^2 |j m (j_1 j_2)\rangle = j_2(j_2+1) |j m (j_1 j_2)\rangle$$

$\Rightarrow$ hledáme vztahy mezi j a j_1, j_2
m a m_1, m_2

↳ tzv. Clebschovy-Gordonovy koeficienty

$m = m_1 + m_2$ $\qquad m_1 \in \{-j_1, -j_1+1, \dots, j_1-1, j_1\}$
$m_2 \in \{-j_2, -j_2+1, \dots, j_2-1, j_2\}$
$m \in \{-(j_1+j_2), \dots, j_1+j_2\}$

pro dané j, m, j_1, j_2

$$|j m\rangle = \sum_i c_i |j_1, i\rangle |j_2, m-i\rangle$$

máme rovnici pro c_i:

$$0 = [j_1(j_1+1) + j_2(j_2+1) - j(j+1) + 2i(m-i)] c_i +$$
$$+ \alpha^+(j_1, i-1)\alpha^-(j_2, m-i+1) c_{i-1} +$$
$$+ \alpha^-(j_1, i+1)\alpha^+(j_2, m-i-1) c_{i+1}$$

kde $\alpha^\pm(j,m) = \sqrt{j(j+1) - m(m\pm1)} = \sqrt{(j\mp m)(j\pm m+1)}$

+ normalizační podmínka: $\sum_i |c_i|^2 = 1$

návod: 1) $j_{min} = |j_1 - j_2|$

$j_{max} = j_1 + j_2$

2) možné stavy:

m	$\vert j_1, m_1\rangle \vert j_2, m_2\rangle$
j_1+j_2	$\vert j_1, j_1\rangle \vert j_2 j_2\rangle$
j_1+j_2-1	$\vert j_1, j_1-1\rangle \vert j_2, j_2\rangle, \vert j_1, j_1\rangle \vert j_2, j_2-1\rangle$
j_1+j_2-2	$\vert j_1, j_1-2\rangle \vert j_2, j_2\rangle, \vert j_1, j_1-1\rangle \vert j_2, j_2-1\rangle, \vert j_1, j_1\rangle \vert j_2, j_2-2\rangle$
⋮	⋮

3) triviální kombinace

4) dobíhající m → jaké j_1, j_2, i

5) vyřešíme c_i (+ norm.)

6) napíšeme LK.

⇒ př. $j_1 = 1/2$ $j_2 = 1/2$ (spin e^- + spin jádra)

1) $\left.\begin{array}{l} j_{min} = 0 \\ j_{max} = 1 \end{array}\right\} \Rightarrow j = 0, 1$

2) možné stavy

m	
$+1$	$\vert +\rangle \vert +\rangle$
0	$\vert +\rangle \vert -\rangle, \vert -\rangle \vert +\rangle$
-1	$\vert -\rangle \vert -\rangle$

3) triviální kombinace: $|1,1\rangle = |+\rangle|+\rangle$
$|1,-1\rangle = |-\rangle|-\rangle$

4) zbylá m:

$m = 0 \Rightarrow$ možné $i = -\frac{1}{2}, +\frac{1}{2}$

$$|\tfrac{1}{2}, i\rangle |\tfrac{1}{2}, m-i\rangle =$$
$$= |\tfrac{1}{2}, i\rangle |\tfrac{1}{2}, -i\rangle \quad i = \pm\tfrac{1}{2}$$

5) spočteme c_i:

$$i = -\tfrac{1}{2}: \left[\tfrac{3}{4} + \tfrac{3}{4} - j(j+1) - \tfrac{1}{2}\right] c_{-\frac{1}{2}} + \alpha^+(\tfrac{1}{2}, -\tfrac{3}{2})\,\alpha^-(\tfrac{1}{2}, \tfrac{3}{2})\, c_{-\frac{3}{2}} + \alpha^-(\tfrac{1}{2}, \tfrac{1}{2})\,\alpha^+(\tfrac{1}{2}, -\tfrac{1}{2})\, c_{\frac{1}{2}} = 0$$

$$[1 - j(j+1)]\, c_{-\frac{1}{2}} + c_{\frac{1}{2}} = 0$$

$$i = +\tfrac{1}{2} \quad \left[\tfrac{3}{4} + \tfrac{3}{4} - j(j+1) - \tfrac{1}{2}\right] c_{\frac{1}{2}} + \alpha^+(\tfrac{1}{2}, -\tfrac{1}{2})\,\alpha^-(\tfrac{1}{2}, \tfrac{1}{2})\, c_{\frac{1}{2}} = 0$$

$$[1 - j(j+1)]\, c_{\frac{1}{2}} + c_{-\frac{1}{2}} = 0$$

normalizace $|c_{-1/2}|^2 + |c_{1/2}|^2 = 0$

$\Rightarrow$ vyřešíme:

a) $j = 1$: $\left.\begin{aligned} -c_{-\frac{1}{2}} + c_{\frac{1}{2}} &= 0 \\ -c_{\frac{1}{2}} + c_{-\frac{1}{2}} &= 0 \end{aligned}\right\} \Rightarrow c_{-1/2} = c_{1/2}$

$$\Rightarrow \underline{\underline{c_{-1/2} = c_{1/2} = \frac{1}{\sqrt{2}}}}$$

b) $j = 0$ $\quad c_{-\frac{1}{2}} + c_{\frac{1}{2}} = 0 \Rightarrow -c_{-1/2} = c_{+1/2}$

$$\Rightarrow \underline{\underline{c_{-1/2} = -c_{1/2} = \frac{1}{\sqrt{2}}}}$$

6) napíšeme LK:

$j=1 \quad |1,0(1/2,1/2)\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle + |-\rangle|+\rangle\right)$

$j=0 \quad |0,0(1/2,1/2)\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle - |-\rangle|+\rangle\right)$

$\Rightarrow$ celkovo:

$$|1,1\,(1/2,1/2)\rangle = |+\rangle|+\rangle$$
$$|1,0,\,(1/2,1/2)\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle + |-\rangle|+\rangle\right)$$
$$|1,-1\,(1/2,1/2)\rangle = |-\rangle|-\rangle$$

$$|0,0(1/2,1/2)\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle|-\rangle - |-\rangle|+\rangle\right)$$

$\boxed{e^- \text{ v } 2p}$

= skladáme $l=1$ a $s=1/2$