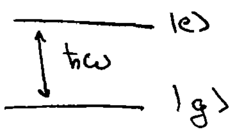
CVIČENÍ

operátor hustoty $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

$\langle\hat{A}\rangle = \mathrm{Tr}(\hat{\rho}\hat{A})$

$\frac{d\hat{\rho}}{dt} = \frac{1}{i\hbar}[\hat{H},\hat{\rho}]$



Ⅰ spontánní emise

Ⅱ $|\psi(t)\rangle = \alpha|e\rangle\otimes|n\rangle + \delta|g\rangle\otimes|n+1\rangle$

$|\psi_A(t=0)\rangle = |e\rangle,\ |\psi_{EM}(t=0)\rangle = \frac{1}{\sqrt{2}}(|p\rangle + |p-1\rangle)$

$\Rightarrow \underline{\underline{|\psi(t)\rangle = \frac{1}{\sqrt{2}}\alpha\,|e\rangle\otimes(|p\rangle+|p-1\rangle) + \frac{1}{\sqrt{2}}\delta\,|g\rangle\otimes(|p\rangle+|p+1\rangle)}}$

$|e\rangle|p\rangle$ a $|g\rangle|p+1\rangle$
$|e\rangle|p-1\rangle$ a $|g\rangle|p\rangle$
mají (po dvojicích) stejnou energii

Ⅲ $|e\rangle\otimes|p-1\rangle,\ |e\rangle\otimes|p\rangle,\ |e\rangle\otimes|p+1\rangle$
$|g\rangle\otimes|p-1\rangle,\ |g\rangle\otimes|p\rangle,\ |g\rangle\otimes|p+1\rangle$

Ⅳ $\hat{\rho} = \sum_i p_i|\psi_i\rangle\langle\psi_i|$

maticové elementy určíme $\langle u_n|\hat{\rho}|u_p\rangle = \langle u_n|\psi\rangle\langle\psi|u_p\rangle$, kde $|u_n\rangle, |u_p\rangle$ jsou naše bázové stavy

→ např. $\underline{\underline{(\langle e|\langle p-1|)\,\hat{\rho}\,(|e\rangle|p-1\rangle)}} = \langle e|\langle p-1|\left[\frac{1}{\sqrt{2}}\alpha|e\rangle(|p\rangle+|p-1\rangle) + \frac{1}{\sqrt{2}}\delta|g\rangle(|p\rangle+|p+1\rangle)\right]\times$

$\times\left[\frac{1}{\sqrt{2}}\alpha^*\langle e|(\langle p|+\langle p-1|) + \frac{1}{\sqrt{2}}\delta^*\langle g|(\langle p|+\langle p+1|)\right]|e\rangle|p-1\rangle =$

$= \Big[\frac{1}{\sqrt{2}}\alpha\ \underline{\langle e|e\rangle}(\langle p-1|p\rangle + \underline{\langle p-1|p-1\rangle}) +$
$+ \frac{1}{\sqrt{2}}\delta\langle e|g\rangle(\langle p-1|p\rangle + \langle p-1|p+1\rangle)\Big]\times$

$\times\Big[\frac{\alpha^*}{\sqrt{2}}\ \underline{\langle e|e\rangle}(\langle p|p-1\rangle + \underline{\langle p-1|p-1\rangle}) +$
$+ \frac{1}{\sqrt{2}}\delta^*\langle g|e\rangle(\langle p|p-1\rangle + \langle p+1|p-1\rangle)\Big] =$

= bázové fce jsou ortogonální a normalizované na 1 (tj. ortonormální)

$\langle e|e\rangle = \langle g|g\rangle = 1 \quad \langle e|g\rangle = \langle g|e\rangle = 0$

$\langle p|p\rangle = 1 \quad \langle p\pm1|p\rangle = 0$

$= \frac{\alpha}{\sqrt{2}}\cdot\frac{\alpha^*}{\sqrt{2}} = \underline{\underline{\frac{1}{2}\alpha\alpha^*}}$

a zcela analogicky pro zbylé členy
(+ můžeme využít symetrie)

$$\Rightarrow \rho = \frac{1}{2}\begin{pmatrix} \alpha\alpha^* & \alpha\alpha^* & 0 & 0 & \alpha\delta^* & \alpha\delta^* \\ \alpha\alpha^* & \alpha\alpha^* & 0 & 0 & \alpha\delta^* & \alpha\delta^* \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha^*\delta & \alpha^*\delta & 0 & 0 & \delta\delta^* & \delta\delta^* \\ \alpha^*\delta & \alpha^*\delta & 0 & 0 & \delta\delta^* & \delta\delta^* \end{pmatrix}$$

V) redukovaná matice hustoty

$$\langle\psi_A|\hat\rho|\psi_A\rangle = \sum_i \langle\psi_{A,}\psi_{EM,i}|\hat\rho|\psi_{A,}\psi_{EM,i}\rangle \equiv \rho_A$$

$$\langle e|\hat\rho_A|e\rangle = \langle e|\langle p-1|\hat\rho|e\rangle|p-1\rangle + \langle e|\langle p|\hat\rho|e\rangle|p\rangle + \langle e|\langle p+1|\hat\rho|e\rangle|p+1\rangle =$$

$$= \frac{1}{2}\alpha\alpha^* + \frac{1}{2}\alpha\alpha^* + 0 =$$

$$= \underline{\underline{\alpha\alpha^*}}$$

a analogicky $\langle e|\hat\rho_A|g\rangle = \underline{\underline{\frac{1}{2}\alpha\delta^*}}$

$\langle g|\hat\rho_A|e\rangle = (\langle e|\hat\rho_A|g\rangle)^* = \underline{\underline{\frac{1}{2}\alpha^*\delta}}$

$\langle g|\hat\rho_A|g\rangle = \underline{\underline{\delta\delta^*}}$

$$\Rightarrow \underline{\underline{\rho_A = \begin{pmatrix} \alpha\alpha^* & \frac{1}{2}\alpha\delta^* \\ \frac{1}{2}\alpha^*\delta & \delta\delta^* \end{pmatrix}}}$$

VI) čistý stav? tj. $\mathrm{Tr}\,\rho_0^2 = 1$?

$$\rho_A^2 = \begin{pmatrix} \alpha\alpha^* & \frac{1}{2}\alpha\delta^* \\ \frac{1}{2}\alpha^*\delta & \delta\delta^* \end{pmatrix}\begin{pmatrix} \alpha\alpha^* & \frac{1}{2}\alpha\delta^* \\ \frac{1}{2}\alpha^*\delta & \delta\delta^* \end{pmatrix} = \begin{pmatrix} \alpha\alpha^*\alpha\alpha^* + \frac{1}{2}\alpha\delta^*\frac{1}{2}\alpha^*\delta & \frac{1}{2}\alpha\alpha^*\alpha\delta^* + \frac{1}{2}\alpha\delta^*\delta\delta^* \\ \frac{1}{2}\alpha^*\delta\alpha\alpha^* + \delta\delta^*\frac{1}{2}\alpha^*\delta & \frac{1}{2}\alpha^*\delta\frac{1}{2}\alpha\delta^* + \delta\delta^*\delta\delta^* \end{pmatrix}$$

$$\mathrm{Tr}\,\rho_A^2 = \alpha\alpha^*\alpha\alpha^* + \frac{1}{4}\alpha\alpha^*\delta\delta^* + \frac{1}{4}\alpha^*\alpha\delta^*\delta + \delta\delta^*\delta\delta^* =$$

$$= (\alpha\alpha^*)^2 + \frac{1}{2}\alpha\alpha^*\delta\delta^* + (\delta\delta^*)^2 =$$

$$= \underbrace{(\alpha\alpha^* + \delta\delta^*)^2}_{=1} - \frac{3}{2}\alpha\alpha^*\delta\delta^* = \underline{\underline{1 - \frac{3}{2}\alpha^*\alpha\delta^*\delta}} < 1 \text{ kromě když } \alpha=0 \text{ nebo } \delta=0$$

↳ nejedná se o čistý stav

VII) $\langle H_A\rangle_0 = \mathrm{Tr}(\rho_{A,0}H_A)$ $(t=0)$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \frac{1}{2}\hbar\omega & 0 \\ 0 & -\frac{1}{2}\hbar\omega \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\hbar\omega & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \mathrm{Tr}(\rho_A H_A) = \underline{\underline{\frac{1}{2}\hbar\omega = \langle H_A\rangle}}$$

VIII) koncový stav $\rho_A(t)$

$$\langle H_A\rangle_t = \mathrm{Tr}(\rho_{A,t}H_A) =$$

$$\begin{pmatrix} \alpha\alpha^* & \frac{1}{2}\alpha\delta^* \\ \frac{1}{2}\alpha^*\delta & \delta\delta^* \end{pmatrix}\begin{pmatrix} \frac{1}{2}\hbar\omega & 0 \\ 0 & -\frac{1}{2}\hbar\omega \end{pmatrix} = \begin{pmatrix} \alpha\alpha^*\frac{1}{2}\hbar\omega & -\frac{1}{2}\alpha\delta^*\frac{\hbar\omega}{2} \\ +\frac{1}{2}\alpha^*\delta\frac{1}{2}\hbar\omega & -\frac{1}{2}\delta\delta^*\hbar\omega \end{pmatrix} \Rightarrow \mathrm{Tr}(\rho_{A,t}H_A) = \underline{\underline{\frac{1}{2}\hbar\omega(\alpha\alpha^* - \delta\delta^*)}}$$

$\Rightarrow H_A$ pro popis nestačí