Povinné DÚ #3

① 2 částice $\rightarrow$ stav $|+z\rangle$

$\searrow$ stav $|+y\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle - i|-z\rangle\right)$

$$\Rightarrow \quad |\psi\rangle = |+z\rangle \otimes \frac{1}{\sqrt{2}}\left(|+z\rangle - i|-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle - i|\uparrow\downarrow\rangle\right)$$

- pst, že získáme stav $|1,0\rangle$?

$$|1,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)$$

$$\mathcal{A} = \langle 1,0|\psi\rangle =$$

$$= \frac{1}{\sqrt{2}}\left(\langle\uparrow\downarrow| + \langle\downarrow\uparrow|\right)\frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle - i|\uparrow\downarrow\rangle\right) =$$

$$= \frac{1}{2}\left(\langle\uparrow\downarrow|\uparrow\uparrow\rangle - i\langle\uparrow\downarrow|\uparrow\downarrow\rangle + \langle\uparrow\downarrow|\uparrow\uparrow\rangle - i\langle\downarrow\uparrow|\uparrow\downarrow\rangle\right)$$

$$\langle\uparrow|\uparrow\rangle_1 \underbrace{\langle\downarrow|\uparrow\rangle_2}_{=0} - i\underbrace{\langle\uparrow|\uparrow\rangle_1}_{=1}\underbrace{\langle\downarrow|\downarrow\rangle_2}_{=1} + \langle\uparrow|\uparrow\rangle_1\underbrace{\langle\downarrow|\uparrow\rangle_2}_{=0} - i\underbrace{\langle\downarrow|\uparrow\rangle_1}_{=0}\underbrace{\langle\uparrow|\downarrow\rangle_2}_{=0}$$

$$= \frac{1}{2}\cdot\left(0 - i + 0 + 0\right) = -i\frac{1}{2}$$

$$\Rightarrow \quad P = |\mathcal{A}|^2 = \left|\left(-i\frac{1}{2}\right)\left(+i\frac{1}{2}\right)\right| = \frac{1}{4}$$

- pst $|+1,+1\rangle$?

$$|+1,+1\rangle = |\uparrow\uparrow\rangle$$

$$\mathcal{A} = \langle\uparrow\uparrow|\frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle - i|\uparrow\downarrow\rangle\right) = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ 1

$\Rightarrow P = \frac{1}{2}$

- pst $|+1,-1\rangle$?

$|+1,-1\rangle = |\downarrow\downarrow\rangle$

$\mathcal{R} = \langle\downarrow\downarrow| \frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle - i|\uparrow\downarrow\rangle) = 0$

$\Rightarrow P = 0$

② - vyjádřit J^2 pomocí $L^2, L_z, L_\pm, S^2, S_z, S_\pm$

$J^2 = (\vec{L}+\vec{S})^2 = L^2 + S^2 + 2\vec{L}\cdot\vec{S}$

$\vec{L}\cdot\vec{S} = \frac{1}{2}(L_+S_- + L_-S_+) + L_zS_z$

$J^2 = L^2 + S^2 + (L_+S_- + L_-S_+) + 2L_zS_z$

- komutátor $[J^2, J_z]$

$[J^2, J_z] = [L_iL_i + S_iS_i + 2L_iS_i, L_j + S_j] =$

$= \underbrace{[L_iL_i, L_j]}_{\substack{=0 \\ [L^2, L_j]=0}} + \underbrace{[S_iS_i, L_j]}_{=0} + 2\underbrace{[L_iS_i, L_j]}_{S_i[L_i,L_j] = S_i i\varepsilon_{ijk}L_k} +$

$+ \underbrace{[S_iS_i, S_j]}_{\substack{=0 \\ [S^2, S_j]=0}} + \underbrace{[L_iL_i, S_j]}_{=0} + 2\underbrace{[L_iS_i, S_j]}_{L_i[S_i,S_j] = L_i i\varepsilon_{ijk}S_k} =$

$= i\varepsilon_{ijk}(L_k + S_k) =$

$$= i \underbrace{\varepsilon_{ijk}}_{A} \underbrace{\left(S_j L_k + L_j S_k \right)}_{\mathrm{SYM}} = \underline{\underline{0}}$$