VLNOVÉ KLUBKO

→ částice pohybující se podél osy x je popsána funkcí

$$\psi(x) = C \exp\left\{ i\frac{p_0 x}{\hbar} \right\} \exp\left\{ -\frac{(x-x_0)^2}{2\sigma^2} \right\}$$

$x_0, p_0 \in \mathbb{R}$
$\sigma > 0$
$C > 0$

(pozn. v Diracově bra-ket notaci
$\psi(x) = \langle x|\psi\rangle$ ... $|\psi\rangle$ = kvantový stav)

1) najít normalizační konstantu C

→ z požadavku $\|\psi\|^2 = \langle\psi|\psi\rangle = \int_{-\infty}^{\infty} \psi^*(x)\psi(x)\,dx$

$$\langle\psi|\psi\rangle = \int_{-\infty}^{+\infty} \psi^*(x)\psi(x)\,dx = \int_{-\infty}^{+\infty} C^* \exp\left\{-i\frac{p_0 x}{\hbar}\right\}\exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\} \cdot C\exp\left\{\frac{ip_0 x}{\hbar}\right\}\exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\} =$$

$$= |C|^2 \int_{-\infty}^{+\infty} \exp\left\{-\frac{(x-x_0)^2}{\sigma^2}\right\} dx$$

nápověda: $\int_{-\infty}^{+\infty} \exp\{-(Ax^2+Bx+C)\}\,dx = \sqrt{\frac{\pi}{A}} \exp\left(-C + \frac{B^2}{4A}\right)$ $\quad A, B, C \in \mathbb{C}$

$$= |C|^2 \int_{-\infty}^{+\infty} \exp\left\{-\left(\frac{1}{\sigma^2}x^2 - \frac{2x_0}{\sigma^2}x + \frac{x_0^2}{\sigma^2}\right)\right\}$$

$$= |C|^2 \sqrt{\frac{\pi}{\frac{1}{\sigma^2}}} \exp\left\{-\frac{x_0^2}{\sigma^2} + \frac{4x_0^2}{\sigma^4} \cdot \frac{1}{4 \cdot \frac{1}{\sigma^2}}\right\}$$

$$= |C|^2 \sigma\sqrt{\pi}\, \underbrace{\exp\{\underbrace{-\tfrac{x_0^2}{\sigma^2} + \tfrac{x_0^2}{\sigma^2}}_{=0}\}}_{=1}$$

$$= |C|^2 \sigma\sqrt{\pi}$$

$$|C|^2\sigma\sqrt{\pi} \stackrel{!}{=} 1 \implies \underline{\underline{|C| = \sqrt[4]{\frac{1}{\pi\sigma^2}}}}$$

$$\langle x|\psi\rangle = \psi(x) = \sqrt[4]{\frac{1}{\pi\sigma^2}} \exp\left\{i\frac{p_0 x}{\hbar}\right\}\exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\}$$

2) označíme $|p\rangle$ rovinnou vlnu hybnosti p

$$\langle x|p\rangle = \frac{1}{\sqrt{2\pi\hbar}} \exp\left\{i\frac{px}{\hbar}\right\}$$

spočítat: $\tilde{\psi}(p) = \langle p|\psi\rangle = \int \underbrace{\langle p|x\rangle\langle x|\psi\rangle}_{\substack{\text{vložíme} \\ \text{rozklad jednotky } \mathbb{1}}} dx = \frac{1}{\sqrt{2\pi\hbar}} \int \exp\left\{-i\frac{px}{\hbar}\right\}\psi(x)\,dx$

$$\tilde{\psi}(p) = \langle p|\psi\rangle = \frac{1}{\sqrt{2\pi\hbar}} \cdot \sqrt[4]{\frac{1}{\pi\sigma^2}} \int \exp\left\{-\frac{ipx}{\hbar}\right\}\exp\left\{i\frac{p_0 x}{\hbar}\right\}\exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\}dx$$

$$= \sqrt[4]{\frac{1}{4\pi^3\hbar^2\sigma^2}} \int \exp\left\{-\left(\frac{1}{2\sigma^2}x^2 + \left(+i\frac{(p-p_0)}{\hbar} - \frac{x_0}{\sigma^2}\right)x + \frac{x_0^2}{2\sigma^2}\right)\right\}dx$$

$$= \sqrt[4]{\frac{1}{4\pi^3\hbar^2\sigma^2}} \left[\sqrt{\frac{\pi}{\frac{1}{2\sigma^2}}} \exp\left\{-\frac{x_0^2}{2\sigma^2} + \frac{\left[\frac{i(p-p_0)}{\hbar} - \frac{x_0}{\sigma^2}\right]^2}{4\,\frac{1}{2\sigma^2}}\right\}\right.$$

$$= \sqrt[4]{\frac{4\sigma^4\pi^2}{4\pi^3\hbar^2\sigma^2}} \exp\left\{-\underline{\frac{x_0^2}{2\sigma^2}} + \frac{1}{2}\left(-\frac{\sigma^2(p-p_0)^2}{\hbar^2} - \cancel{\sigma^2}\frac{2i(p-p_0)x_0}{\cancel{\sigma^2}\hbar} + \underline{\frac{\sigma^2 x_0^2}{\sigma^4}}\right)\right\}$$

$$= \sqrt[4]{\frac{\sigma^2}{\pi\hbar^2}} \exp\left\{-\frac{\sigma^2(p-p_0)^2}{2\hbar^2} - \frac{i(p-p_0)x_0}{\hbar}\right\}$$

$$= \sqrt[4]{\frac{\sigma^2}{\pi\hbar^2}} \exp\left\{-\frac{1}{2}\frac{(p-p_0)^2}{(\frac{\hbar}{\sigma})^2}\right\} \exp\left\{-\frac{i}{\hbar}(p-p_0)x_0\right\} = \Psi(p)$$

kolik je $\tilde{P}(p) = |\Psi(p)|^2 = ?$

$$|\Psi(p)|^2 = \frac{\sigma}{\hbar\sqrt{\pi}} \exp\left\{-\frac{(p-p_0)^2}{(\frac{\hbar}{\sigma})^2}\right\}$$ → pravděpodobnost ~~hustota pst.~~, že částice/vlna má hybnost p

3) co se stane z $|\psi\rangle$ v limitách $\sigma \to \infty$, $\sigma \to 0$?

$$\langle x|\psi\rangle = \psi(x) = C\exp\left\{i\frac{p_0 x}{\hbar}\right\}\exp\left\{-\frac{(x-x_0)^2}{2\sigma^2}\right\} \xrightarrow{\sigma\to 0} \delta(x-x_0)$$ Diracova distribuce

$$\xrightarrow{\sigma\to +\infty} \psi(x) \approx C\exp\left\{\frac{i p_0 x}{\hbar}\right\}$$ rovinná vlna

4) výpočet středních hodnot:

$\langle x\rangle = x_0$ $\qquad$ $\langle p\rangle = p_0$

$\langle x^2\rangle = \frac{1}{2}\sigma^2 + x_0^2$ $\qquad$ $\langle p^2\rangle = \frac{\hbar^2}{2\sigma^2} + p_0^2$

$\Delta x = \sqrt{\langle (x-\langle x\rangle)^2\rangle} = \frac{\sigma}{\sqrt{2}}$ $\qquad$ $\Delta p = \frac{1}{\sqrt{2}}\frac{\hbar}{\sigma}$

} dopočítat! (DÚ)

a kolik je $\Delta x \cdot \Delta p = \frac{\hbar}{2}$

$$\underline{\underline{\langle x\rangle}} = \langle\psi|\hat{x}|\psi\rangle = \int_{-\infty}^{+\infty}\langle\psi|x\rangle\langle x|\hat{x}|\psi\rangle dx = \int_{-\infty}^{+\infty}\psi^*(x)\, x\, \psi(x)\, dx = \int_{-\infty}^{+\infty} x\,|\psi(x)|^2 dx =$$

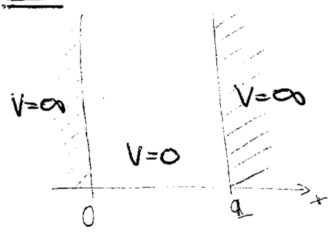
$$= \int_{-\infty}^{+\infty} x\, C^2 \exp\left\{-\frac{(x-x_0)^2}{\sigma^2}\right\}dx = C^2 x_0\sqrt{\pi\sigma^2} = \underline{\underline{x_0}}$$

$$\int_{-\infty}^{+\infty} x\exp\left\{-\frac{(x-x_0)^2}{\sigma^2}\right\}dx = \underbrace{\int_{-\infty}^{+\infty}(x-x_0)\exp\{\dots\}dx}_{\substack{\text{lichá fce} \\ \Rightarrow\, = 0}} + \int_{-\infty}^{+\infty} x_0\exp\{\dots\}dx = x_0\int_{-\infty}^{+\infty}\exp\left\{\frac{-(x-x_0)^2}{\sigma^2}\right\}dx = x_0\sqrt{\pi\sigma^2}$$

$$\langle x^2\rangle = \int_{-\infty}^{+\infty} x^2|\psi(x)|^2 dx = \cdots$$

ČÁSTICE V NEKONEČNÉ POTENCIÁLOVÉ JÁMĚ

1D



$V(x)=0$ pro $x\in(0,L)$

$V=\infty$ pro $x\in(-\infty,0)\cup(L,+\infty)$

→ částice se může vyskytovat pouze v $(0,L)$, mimo tento interval ne, tj. $\psi(x)=0$ pro $x\notin(0,L)$

→ pozn.: bude na přednášce:

postulát QM: systém popsán vln. fcí ψ

Schrödinger: $H\psi=E\psi$ ~ bezčasová Schrödingerova rovnice

E = energie, kt. hledáme

ψ = vln. fce, —"—

H = Hamiltonián systému, zadán

⇒ pro náš případ:

$$H=-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}=E\psi(x) \quad \text{v intervalu } x\in(0,a)$$

$$H=T+V \quad T=\text{kinetická energie}=\frac{\hat{p}^2}{2m}=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

$$\hat{p}=-i\hbar\frac{d}{dx}$$

V = potenciální energie $=0$

→ řešení přes charakteristický polynom

$$-\frac{\hbar^2}{2m}\psi''(x)-E\psi(x)=0$$

$$\psi''(x)+\frac{E2m}{\hbar^2}\psi(x)=0$$

$$\psi''(x)+k^2\psi(x)=0 \qquad \psi(x)=Ae^{i\lambda x}$$

⇒ char. poly: $\lambda^2+\underbrace{\frac{2mE}{\hbar^2}}_{k^2}=0$ ⇒ $\lambda=\pm ik$

⇒ obecné řešení

$$\psi(x)=Ae^{ikx}+Be^{-ikx}$$

a máme okrajové podmínky: $\psi(0)=\psi(L)=0$

→ $\psi(0)=A+B=0$ ⇒ $A=-B$

$$\sin x=\frac{1}{2i}(e^{ix}-e^{-ix})$$

$$\cos x=\frac{1}{2}(e^{ix}+e^{-ix})$$

$$\psi(x)=A(e^{ikx}-e^{-ikx})=C\sin(kx)$$

→ $\psi(L)=C\sin(kL)=0$

⇒ $kL=\pi n \quad n=1,2\dots$

⇒ $k_n=\frac{\pi}{L}n$

vlnový vektor

$$\Rightarrow E=\frac{\hbar^2}{2m}k^2=\frac{\hbar^2\pi^2n^2}{2mL^2}$$

normovací konstanta

$$\psi_n(x)=N\sin\left(\frac{\pi x n}{L}\right)$$

⇒ vlnový vektor i odpovídající energie jsou kvantovány

→ normovací konstantu dopočteme z požadavku normalizace:

$$\int_0^L |N\psi(x)|^2 dx = \int_0^L |N|^2 \sin^2\frac{\pi x n}{L} \stackrel{!}{=} 1$$

$\cos 2x = \cos^2 x - \sin^2 x$ ⇔ ~~$\sin^2 x =$~~ $= 1 - 2\sin^2 x$

→ $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

$$= \int_0^L |N|^2 \frac{1}{2}\left(1 - \cos\frac{2\pi n x}{L}\right) dx$$

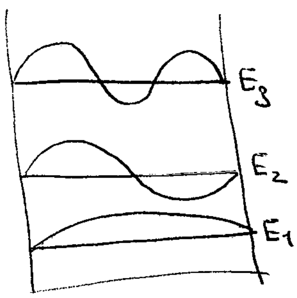
$$= |N|^2 \frac{1}{2}\left[x - \frac{L}{2\pi n x}\sin\frac{2\pi n x}{L}\right]_0^L$$

$$= |N|^2 \frac{1}{2} L \quad \rightarrow \quad \underline{\underline{|N| = \sqrt{\frac{2}{L}}}} \cdot e^{i\alpha}$$

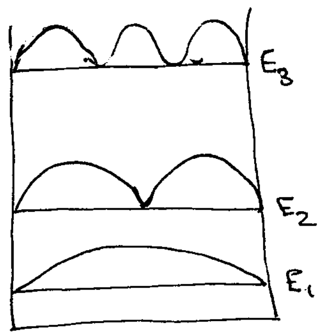
← fáze ⇒ volnost ve volbě
⇒ zpravidla volíme $=1$

$$\Rightarrow \underline{\underline{\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi n x}{L}\right)}}$$

⇒



a pravděpodobnosti výskytu:



3D

potenciál $V(x,y,z) = 0$ pro $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$ $a \times b \times c$ ... rozměry jámy

jinak $V(x,y,z) \rightarrow \infty$

⇒ Schrödingerova rovnice

$$-\frac{\hbar^2}{2m}\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)\psi(x,y,z) = E\psi(x,y,z)$$

⇒ separace $\psi(x,y,z) = \psi_x(x)\,\psi_y(y)\,\psi_z(z)$

⇒ $E = E_x + E_y + E_z$

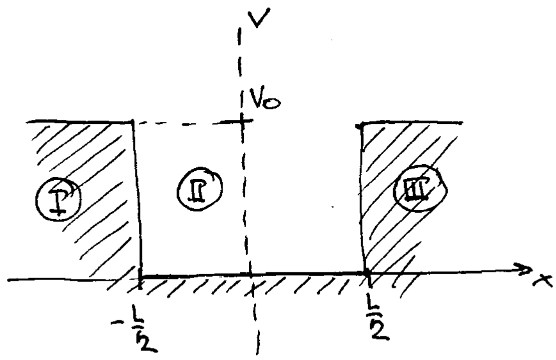
⇒ $-\frac{\hbar^2}{2m}\frac{d^2}{dx_i^2}\psi_i(x_i) = E_i \psi_i(x_i)$

⇝ použitím výsledků pro 1D: $\psi_{lmn}(x,y,z) = \sqrt{\frac{8}{abc}} \sin\frac{\pi x l}{a} \sin\frac{\pi y m}{b} \sin\frac{\pi z n}{c}$, $l,m,n = 1,2,\ldots$

→ degenerované hladiny → např. $E_{112}, E_{121}, E_{211}$

↳ základní hladina E_{111} nedegenerovaná a

POTENCIÁLOVÁ JÁMA KONEČNÉ HLOUBKY → najít diskrétní hladiny



$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + (E-V(x))\psi(x) = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V(x))}{\hbar^2}\psi(x) = 0$$

Schr.
→ vyřešíme zvlášť a použijeme „sešívací podmínky": spojitost ψ a ψ'

$$\psi_I(-\tfrac{L}{2}) = \psi_{II}(-\tfrac{L}{2}) \qquad \psi_{II}(\tfrac{L}{2}) = \psi_{III}(\tfrac{L}{2})$$

$$\psi_I'(-\tfrac{L}{2}) = \psi_{II}'(-\tfrac{L}{2}) \qquad \psi_{II}'(L/2) = \psi_{III}'(L/2)$$

+ okrajové podmínky:

$$\lim_{x\to-\infty}\psi_I(x) = 0$$

$$\lim_{x\to+\infty}\psi_{III}(x) = 0$$

→ obecné řešení Schr. rce $\quad \frac{d^2}{dx^2}\psi(x) + \frac{2m(E-V(x))}{\hbar^2}\psi(x) = 0$

↳ pro $E<V_0$: $\quad \frac{d^2\psi_I}{dx^2} - \alpha^2\psi_I = 0 \qquad \alpha = \sqrt{\frac{2m(V_0-E)}{\hbar^2}} > 0$

$$\psi_I(x) = A_I e^{\alpha x} + B_I e^{-\alpha x} \quad \text{... obecně} \qquad A_I, B_I \in \mathbb{C}$$

ale okraj. podm. $\lim_{x\to-\infty}\psi_I(x) = 0 \Rightarrow B_I = 0$

$$\psi_I(x) = A_I e^{\alpha x}$$

a podobně pro $\psi_{III}(x) = B_{III} e^{-\alpha x}$

↳ pro (II): $\quad \frac{d^2}{dx^2}\psi_{II}(x) + k^2\psi_{II}(x) = 0 \qquad k = \sqrt{\frac{2mE}{\hbar^2}} > 0$

$$\Rightarrow \psi_{II}(x) = A_{II}\sin(kx) + B_{II}\cos(kx) \qquad A_{II}, B_{II} \in \mathbb{C}$$

potenciál symetrický → lze ukázat, že vl. fce hamiltoniánu musejí být sudé anebo liché → viz Stála UdKM

$$\psi_{II}^S = B_{II}\cos(kx) \qquad \psi_{II}^L = A_{II}\sin(kx)$$

1) sudé řešení

$$\psi_I(x) = A_I e^{\alpha x} \qquad x \le -\tfrac{L}{2}$$

$$\psi_{II}(x) = B_{II}\cos(kx) \qquad |x| \le \tfrac{L}{2}$$

$$\psi_{III}(x) = B_{III} e^{-\alpha x} \qquad x \ge \tfrac{L}{2}$$

vzhledem k symetrii $A_I = B_{III}$

②

→ sešívací podm.: $\psi_{II}(L/2)=\psi_{III}(L/2)$: $B_{II}\cos(k\frac{L}{2}) = A\exp\{-\frac{\alpha L}{2}\}$

$\psi_{II}'(L/2)=\psi_{III}'(L/2)$: $-\sin(k\frac{L}{2})B_{II}k = -\alpha A\exp\{-\frac{\alpha L}{2}\}$ $\Big\}\Rightarrow$

$$\Rightarrow \quad A e^{-\frac{\alpha L}{2}} - B_{II}\cos(\tfrac{kL}{2}) = 0$$
$$\alpha A e^{-\frac{\alpha L}{2}} - kB_{II}\sin(\tfrac{kL}{2}) = 0$$
} řešíme pro A, B_{II}, $\underline{k}$ je parametr ($k\to E$)

$\det(\dots) \overset{!}{=} 0$ (aby soustava lin. rovnic měla netriviální řešení)

$$-Ae^{-\frac{\alpha L}{2}}kB_{II}\sin(\tfrac{kL}{2}) + \alpha A e^{-\frac{\alpha L}{2}}B_{II}\cos(\tfrac{kL}{2}) = 0$$

a hledáme $\underline{k}$

$$B_{II}Ae^{-\frac{\alpha L}{2}}\left(-k\sin(\tfrac{kL}{2}) + \alpha\cos(\tfrac{kL}{2})\right) = 0$$

$$-k\sin(\tfrac{kL}{2}) + \alpha\cos(\tfrac{kL}{2}) = 0$$

$$\underline{\mathrm{tg}(\tfrac{kL}{2})} = \frac{\alpha}{k} = \sqrt{\frac{V_0-E}{E}} = \underline{\sqrt{\frac{2mV_0}{\hbar^2k^2}-1}} = \underline{\frac{\alpha\hbar}{\sqrt{2mE}}}$$

$$kL = 2\,\mathrm{arctg}(\sqrt{\dots}) + 2n\pi = 2\,\mathrm{arctg}\left(\frac{\alpha\hbar}{\sqrt{2mE}}\right) + 2n\pi,\ n\in\mathbb{Z}$$

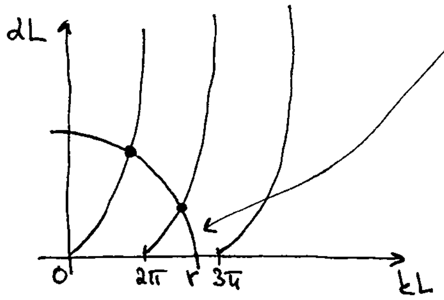
a také platí $k^2+\alpha^2 = \frac{2m}{\hbar^2}V_0$

$$(kL)^2 + (\alpha L)^2 = \frac{2m}{\hbar}V_0L^2 \quad \sim \text{kružnice } x^2+y^2=r^2$$

$$\frac{2mV_0-\hbar^2k^2}{\hbar^2k^2} = \frac{2mV_0-2mV_0+\hbar^2\alpha^2}{2mE} = \frac{\alpha^2\hbar^2}{2mE}.$$

$$k^2 = \frac{2m}{\hbar^2}V_0 - \alpha^2$$
$$k^2 = \frac{2m}{\hbar^2}E$$

⇒ grafické řešení



počet průsečíků = počet k = počet diskrétních energ. hladin v jámě

2) lichá řešení

$\psi_I(x) = A_I e^{\alpha x}$ $\quad x \le -\frac{L}{2}$

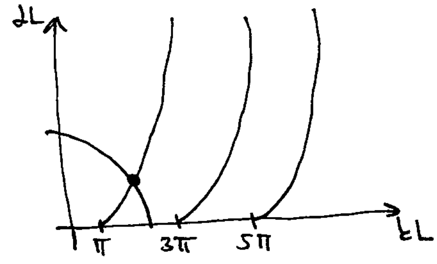
$\psi_{II}(x) = A_{II}\sin(kx)$ $\quad |x| < \frac{L}{2}$

$\psi_{III}(x) = B_{III}e^{-\alpha x}$ $\quad x \ge \frac{L}{2}$

postupujeme zcela analogicky a dostaneme:

$$\underline{-\mathrm{cotg}(\tfrac{kL}{2})} = \frac{\alpha}{k} = \sqrt{\frac{2mV_0}{\hbar^2k^2}-1} = \underline{\frac{\alpha\hbar}{\sqrt{2mE}}}$$

$$(kL)^2 + (\alpha L)^2 = \frac{2m}{\hbar}V_0L^2$$



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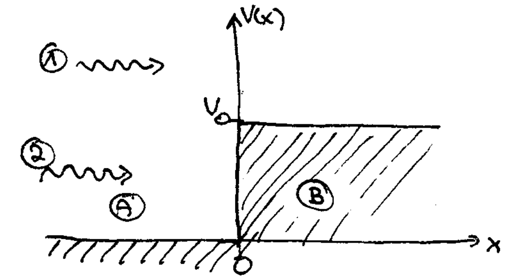
(*) ⇒ řešení spojíme dohromady:

↳ počet diskrétních hladin N záleží na poloměru čtvrtkružnice

$$r = \sqrt{\frac{2m_0V_0L^2}{\hbar^2}}$$

$$(N-1)\pi < r = \sqrt{\frac{2m_0V_0L^2}{\hbar^2}} < N\pi$$

POTENCIÁLOVÝ SCHOD



1 $E > V_0$: částečný odraz

$$H\psi(x) = E\psi(x) \qquad H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

→ A: $\psi_A''(x) + \frac{2mE}{\hbar^2}\psi_A(x) = 0 \Rightarrow \boxed{\psi_A(x) = A_1 e^{ik_Ax} + A_2 e^{-ik_Ax}}$ $\quad k_A = \sqrt{\frac{2mE}{\hbar^2}}$

přilétavající vlna

odražená vlna

→ B: $\psi_B''(x) + \frac{2m(E-V_0)}{\hbar^2}\psi_B(x) = 0 \Rightarrow \boxed{\psi_B(x) = B_1 e^{ik_Bx} + B_2 e^{-ik_Bx}}$ $\quad k_B = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$

prošlá vlna

$\doteq 0$

protože ~~vlna~~ letí zleva doprava

↳ A_1, A_2, B_1 z požadavku spojitosti $\psi(x), \psi'(x)$:
(sešívací podmínky)

$\psi_A(0) = \psi_B(0)$: $\quad A_1 + A_2 = B_1$

$\psi_A'(0) = \psi_B'(0)$: $\quad ik_AA_1 - ik_AA_2 = ik_BB_1$

$$k_AA_1 - k_AA_2 = k_BB_1$$

$k_AA_1 - k_AA_2 = k_BB_1 = k_B(A_1 + A_2)$

$A_1(k_A - k_B) = A_2(k_B + k_A)$

$$\frac{A_1}{A_2} = \frac{k_B + k_A}{k_A - k_B}$$

$k_AA_1 - k_A(\overbrace{B_1 - A_1}^{A_2}) = k_BB_1$

$2k_AA_1 = B_1(k_B + k_A)$

$$\frac{B_1}{A_1} = \frac{2k_A}{k_A + k_B}$$

$$\frac{B_1}{A_2} = \frac{\frac{A_1}{A_2}}{\frac{A_1}{B_1}} = \frac{2k_A}{k_A \mp k_B}$$

↳ trasmisní koeficient

$T = \frac{\text{prošlý tok}}{\text{dopadající tok}} \qquad \text{tok} = j(x) = -\frac{i\hbar}{2m}\left(\psi^*(x)\psi'(x) - \psi'^*(x)\psi(x)\right)$

↳ prošlý tok: $\psi_B(x) \Rightarrow j(x) = -\frac{i\hbar}{2m}\left(B_1^* e^{-ik_Bx}(+ik_B)B_1 e^{+ik_Bx} - (-ik_B)B_1^* e^{-ik_Bx} B_1 e^{+ik_Bx}\right)$

$$= -\frac{i\hbar}{2m}|B_1|^2 \cdot 2ik_B =$$

$$= \frac{\hbar k_B}{m}|B_1|^2$$

↳ dopadající tok: $\approx A_1 e^{ik_A x}$ $\Rightarrow$ $j_D(x) = |A_1|^2 \frac{\hbar k_{A_1}}{m}$

↳ odražený tok: $\approx A_2 e^{-ik_A x}$ $\Rightarrow$ $j_R(x) = |A_2|^2 \frac{\hbar k_{A_2}}{m}$

$\Rightarrow$ transmise $\underline{T} = \dfrac{|B_1|^2 \frac{\hbar k_B}{m}}{|A_1|^2 \frac{\hbar k_A}{m}} = \dfrac{k_B}{k_A}\left|\dfrac{B_1}{A_1}\right|^2 = \dfrac{k_B}{k_A}\dfrac{4k_A^2}{(k_B+k_A)^2} = \underline{\underline{\dfrac{4k_A k_B}{(k_B+k_A)^2}}}$

reflexe $\underline{R} = \dfrac{|A_2|^2 \frac{\hbar k_A}{m}}{|A_1|^2 \frac{\hbar k_A}{m}} = \left|\dfrac{A_2}{A_1}\right|^2 = \underline{\underline{\dfrac{(k_A-k_B)^2}{(k_A+k_B)^2}}}$

$$\underline{\underline{R+T}} = \frac{(k_A-k_B)^2}{(k_A+k_B)^2} + \frac{4k_A k_B}{(k_A+k_B)^2} = \frac{(k_A+k_B)^2}{(k_A+k_B)^2} = \underline{\underline{1}}$$

(2) $E < V_0$: totální odraz

$\psi_A(x) = A_1 e^{ik_A x} + A_2 e^{-ik_A x}$ $\qquad k_A = \sqrt{\frac{2mE}{\hbar^2}}$

$\psi_B(x) = B_1 e^{ik_B x} + B_2 e^{-ik_B x}$ $\qquad k_B = \sqrt{\frac{2m}{\hbar^2}(E-V_0)}$

$\Rightarrow \rho_B = -i k_B = -i\sqrt{2m(E-V_0)/\hbar^2} = \sqrt{2m(V_0-E)/\hbar^2}$

$= B_1 e^{-\rho_B x} + B_2 e^{+\rho_B x}$

$\longrightarrow \qquad \longleftarrow$

$B_2 = 0$ (svazek přichází zleva)

částice šíří se $\rightarrow$ proniká do bariéry! $\rightarrow$ ale nemůže se protunelovat (bariéra nekonečná) $\Rightarrow$ $\Rightarrow$ odrazí se zpět (evanescentní vlna nenese energii)

↳ A_1, A_2, B_1 ze sešívacích podmínek:

$\psi_A(0) = \psi_B(0)$: $\quad A_1 + A_2 = B_1$

$\psi'_A(0) = \psi'_B(0)$: $\quad ik_A A_1 - ik_A A_2 = -\rho_B B_1$

$k_A A_1 - k_A A_2 = i\rho_B B_1$

$k_A A_1 - k_A A_2 = i\rho_B (A_1 + A_2)$

$(k_A - i\rho_B) A_1 = A_2 (k_A + i\rho_B)$

$\dfrac{A_1}{A_2} = \dfrac{k_A + i\rho_B}{k_A - i\rho_B}$

$k_A A_1 - k_A (B_1 - A_1) = i\rho_B B_1$

$2k_A A_1 = (k_A + i\rho_B) B_1$

$\dfrac{B_1}{A_1} = \dfrac{2k_A}{k_A + i\rho_B}$

↳ reflektivita:

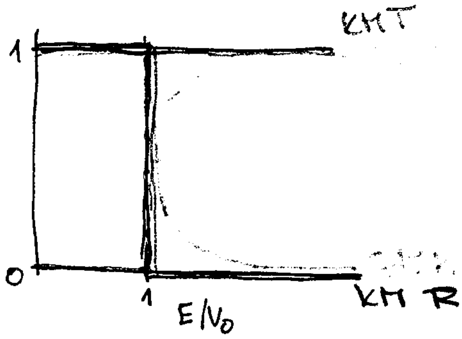
$$\underline{\underline{R}} = \left|\frac{A_2}{A_1}\right|^2 = \left|\frac{k_A - i\rho_B}{k_A + i\rho_B}\right|^2 = \frac{(k_A - i\rho_B)(k_A + i\rho_B)}{(k_A + i\rho_B)(k_A - i\rho_B)} = \underline{\underline{1}}$$

$|z|^2 = z \cdot z^*$

$$j(x) = \frac{-i\hbar}{2m}\left(\psi^*(x)\psi'(x) - \psi^{*\prime}(x)\psi(x)\right)$$

$$\underline{\underline{j_T(x)}} = -\frac{i\hbar}{2m}\left[B_1^* e^{-\rho_B x} B_1(-\rho_B) e^{-\rho_B x} - B_1^*(-\rho_B) e^{-\rho_B x} B_1 e^{-\rho_B x}\right]$$

$$= -\frac{i\hbar}{2m}\left(B_1^* B_1(-\rho_B) - B_1^* B_1(-\rho_B)\right) e^{-2\rho_B x} = \underline{\underline{0}}$$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