Povinné DÚ #5

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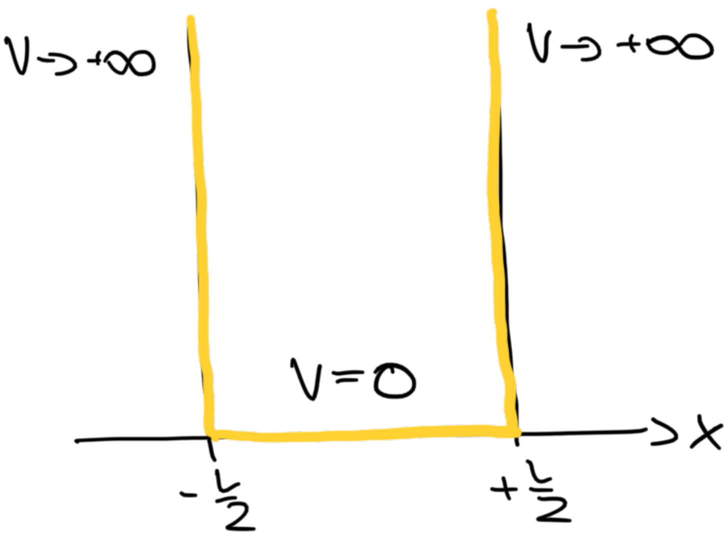

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) = E\psi(x)$$

$$\frac{d^2}{dx^2}\psi(x) + \underbrace{\frac{2mE}{\hbar^2}}_{k^2}\psi(x) = 0$$

$$\psi(x) = Ae^{+ikx} + Be^{-ikx}$$

okrajové podmínky:

$$\psi\left(-\frac{L}{2}\right) = 0$$

$$\psi\left(+\frac{L}{2}\right) = 0$$

$$\psi\left(-\frac{L}{2}\right) = Ae^{-ik\frac{L}{2}} + Be^{+ik\frac{L}{2}} = 0$$

$$\psi\left(+\frac{L}{2}\right) = Ae^{+ik\frac{L}{2}} + Be^{-ik\frac{L}{2}} = 0$$

$$\Big(e^{-ik\frac{L}{2}} \quad e^{+ik\frac{L}{2}} \Big) \Big(A \Big)$$

$$\begin{pmatrix} & \\ e^{+ik\frac{L}{2}} & e^{-ik\frac{L}{2}} \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = 0$$

$$\det(\dots) = e^{-ikL} - e^{ikL} \stackrel{!}{=} 0$$

$$e^{-ikL} = e^{+ikL}$$

$$1 = e^{2ikL}$$

$$2ikL = 2in\pi \quad n \in \mathbb{N}$$

$$\underline{\underline{k_n = \frac{\pi n}{L}}}$$

energie kvantovaná

$$B = e^{in\pi} A = -(-1)^n A$$

$\Rightarrow$ vlastní fce: $\psi_n(x) = A_n \left(e^{ik_n x} - (-1)^n e^{-ik_n x} \right)$

$$= A_n \left(e^{\frac{in\pi}{L} x} - (-1)^n e^{-\frac{in\pi}{L} x} \right)$$

$\Rightarrow$ n sudé $\psi_n(x) = N \sin\left(\frac{\pi n}{L} x\right)$

n liché $\psi_n(x) = N \cos\left(\frac{\pi n}{L} x\right)$

z normalizace $\langle \psi | \psi \rangle \stackrel{!}{=} 1 \Rightarrow N = \sqrt{\frac{2}{L}}$

$$\Rightarrow \psi_n = \begin{cases} \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right) & n \text{ sudé} \\ \sqrt{\frac{2}{L}} \cos\left(\frac{n\pi}{L} x\right) & n \text{ liché} \end{cases}$$

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

(2) $\psi(x) = N \left[\sin\left(\frac{3\pi x}{L}\right) - \sin\left(\frac{\pi x}{L}\right) \right]$

$\Rightarrow$ normalizace

$$\langle\psi|\psi\rangle \overset{!}{=} 1$$

$$\int_0^L dx\, |N|^2 \left[\sin\left(\frac{3\pi x}{L}\right) - \sin\left(\frac{\pi x}{L}\right)\right]^2 =$$

$$= \int_0^L dx\, |N|^2 \left[\sin^2\left(\frac{3\pi x}{L}\right) - 2\sin\left(\frac{3\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right) + \sin^2\left(\frac{\pi x}{L}\right)\right] =$$

$$= |N|^2\left(\frac{L}{2} + 0 + \frac{L}{2}\right) = LN^2 \Rightarrow \underline{\underline{N = \frac{1}{\sqrt{L}}}}$$

$\Rightarrow \langle H\rangle_\psi = \langle\psi|H|\psi\rangle$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|3\rangle - |1\rangle\right)$$

$|1\rangle$ = první stav s $E_1 = \frac{\hbar^2\pi^2}{2mL^2}1^2$

$|3\rangle$ = třetí stav s $E_3 = \frac{\hbar^2\pi^2}{2mL^2}3^2$

$$\langle H\rangle_\psi = \frac{1}{\sqrt{2}}\left(\langle 1| - \langle 3|\right) H \frac{1}{\sqrt{2}}\left(|1\rangle - |3\rangle\right) =$$

$$= \frac{1}{2}\Big(\underbrace{\langle 1|H|1\rangle}_{E_1\langle 1|1\rangle = E_1} - \underbrace{\langle 3|H|1\rangle}_{=0} - \underbrace{\langle 1|H|3\rangle}_{=0} + \underbrace{\langle 3|H|3\rangle}_{E_3\langle 3|3\rangle = E_3}\Big) =$$

$\langle 3|1\rangle = 0$ $\quad$ $\langle 1|3\rangle = 0$

$$= \frac{1}{2}\left(E_1 + E_3\right) =$$

$$= \underline{\underline{\frac{\hbar^2\pi^2}{4mL^2}(1+9) = \frac{5}{2}\frac{\hbar^2\pi^2}{mL^2} = \langle H\rangle_\psi}}$$

$\Rightarrow$ pst $(0, \frac{L}{2})$

$$\int_0^{L/2} |\psi(x)|^2 dx = \int_0^{L/2} \frac{1}{L}\left(\sin^2\left(\frac{3\pi x}{L}\right) + \sin^2\left(\frac{\pi x}{L}\right) - 2\sin\left(\frac{3\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)\right) dx =$$

$\frac{L}{4}$ $\quad$ $\frac{L}{4}$

$$= \frac{1}{L}\left(\frac{L}{4}+\frac{L}{4}\right) = \underline{\underline{\frac{1}{2}}}$$

$\Rightarrow$ pst $\left(\frac{L}{4}, \frac{3}{4}L\right)$

$$\int_{L/4}^{3L/4} |\psi(x)|^2 dx = \int_{L/4}^{3L/4} \frac{1}{L}\left(\sin^2\left(\frac{3\pi x}{L}\right) + \sin^2\left(\frac{\pi x}{L}\right) - 2\sin\left(\frac{3\pi x}{L}\right)\sin\left(\frac{\pi x}{L}\right)\right) dx =$$

$$= \frac{1}{2} + \frac{4}{3\pi} \approx 0{,}92$$

$$\Rightarrow \underline{\underline{92\%}}$$