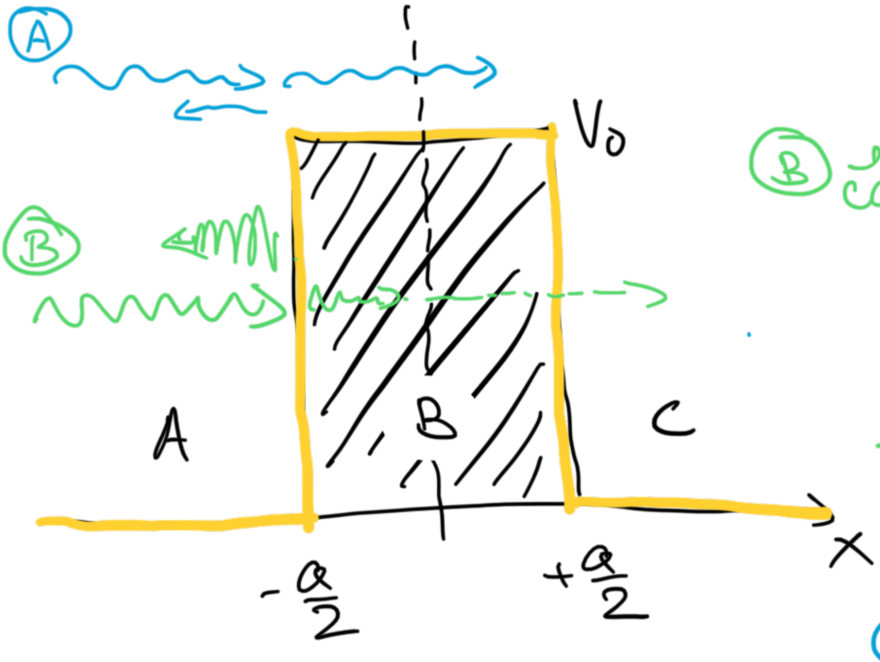
POTENCIÁLOVÝ VAL



(B) částice přilétá z $-\infty$ a má energii $E<V_0$

$\rightarrow$ KM: odrazí se zpět

$\rightarrow$ QM: s nějakou pst. bariérou <u>projde</u>

(A) částice má energii $E>V_0$

$\rightarrow$ KM: vše projde

$\rightarrow$ QM: nenulová pst odrazu

(A) částice $E>V_0$

$$\psi_A(x) = A_1 e^{ik_A x} + A_2 e^{-ik_A x} \qquad x \leq -\frac{a}{2}$$

$$\psi_B(x) = B_1 e^{ik_B x} + B_2 e^{-ik_B x} \qquad -\frac{a}{2} \leq x \leq \frac{a}{2}$$

$$\psi_C(x) = C_1 e^{ik_C x} + C_2 e^{-ik_C x} \qquad \frac{a}{2} \leq x$$

$$k_A = k_C = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k_B = \sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

částice přilétá z $-\infty \Rightarrow C_2 = 0$

sešívací podmínky:

- $\psi_A(-\frac{a}{2}) = \psi_B(-\frac{a}{2})$ a $\psi'_A(-\frac{a}{2}) = \psi'_B(-\frac{a}{2})$

$$\Rightarrow A_1 e^{-ik_A\frac{a}{2}} + A_2 e^{+ik_A\frac{a}{2}} = B_1 e^{-ik_B\frac{a}{2}} + B_2 e^{+ik_B\frac{a}{2}} \quad (1)$$

$$\Rightarrow ik_A A_1 e^{-ik_A\frac{a}{2}} - ik_A A_2 e^{+ik_A\frac{a}{2}} =$$
$$= ik_B B_1 e^{-ik_B\frac{a}{2}} - ik_B B_2 e^{+ik_B\frac{a}{2}} \quad (2)$$

- $\psi_B(\frac{a}{2}) = \psi_C(\frac{a}{2})$ a $\psi'_B(\frac{a}{2}) = \psi'_C(\frac{a}{2})$

$$\Rightarrow B_1 e^{ik_B\frac{a}{2}} + B_2 e^{-ik_B\frac{a}{2}} = C_1 e^{ik_C\frac{a}{2}} \quad (3)$$

$$\Rightarrow ik_B B_1 e^{ik_B\frac{a}{2}} - ik_B B_2 e^{-ik_B\frac{a}{2}} = ik_C C_1 e^{ik_C\frac{a}{2}} \quad (4)$$

⇒ chceme najít vztah mezi $A_1, A_2, B_1, B_2, C_1, C_2$

⇒ z toho transmise T a reflexe R

$$T = \frac{\text{prošlý tok}}{\text{dopadající tok}} = \left|\frac{C_1}{A_1}\right|^2$$

⇒ chceme najít vztah mezi C_1 a A_1

$$(1) + \frac{1}{ik_B}(2): \left(1+\frac{k_A}{k_B}\right) A_1 e^{-ik_A\frac{a}{2}} + \left(1-\frac{k_A}{k_B}\right) A_2 e^{+ik_A\frac{a}{2}} =$$
$$= 2B_1 e^{-ik_B\frac{a}{2}}$$

$$(1) - \frac{1}{ik_B}(2): \left(1-\frac{k_A}{k_A}\right) A_1 e^{-ik_A\frac{a}{2}} + \left(1+\frac{k_A}{k_B}\right) A_2 e^{+ik_A\frac{a}{2}} =$$
$$= 2B_2 e^{+ik_B\frac{a}{2}}$$

$$(3) + \frac{1}{ik_B}(4): 2B_1 e^{ik_B\frac{a}{2}} = \left(1+\frac{k_C}{k_B}\right) C_1 e^{ik_C\frac{a}{2}}$$

$$(3) - \frac{1}{ik_B}(4): 2B_2 e^{-ik_B\frac{a}{2}} = \left(1-\frac{k_C}{\ }\right) C_1 e^{+ik_C\frac{a}{2}}$$

k_B

$$\left(1+\frac{k_A}{k_B}\right)A_1 e^{-ik_A\frac{a}{2}} + \left(1-\frac{k_A}{k_B}\right)A_2 e^{+ik_A\frac{a}{2}} = \left(1+\frac{k_C}{k_B}\right)C_1 e^{ik_C\frac{a}{2}} e^{-ik_B a}$$

$$\left(1-\frac{k_A}{k_B}\right)A_1 e^{-ik_A\frac{a}{2}} + \left(1+\frac{k_A}{k_B}\right)A_2 e^{+ik_A\frac{a}{2}} = \left(1-\frac{k_C}{k_B}\right)C_1 e^{ik_C\frac{a}{2}} e^{ik_B a}$$

$$\left(1-\frac{k_A}{k_B}\right)A_2 e^{+ik_A\frac{a}{2}} = -\left(1+\frac{k_A}{k_B}\right)A_1 e^{-ik_A\frac{a}{2}} + \left(1+\frac{k_C}{k_B}\right)C_1 e^{ik_C\frac{a}{2}} e^{-ik_B \dots}$$

$$\left(1-\frac{k_A}{k_B}\right)A_1 e^{-ik_A\frac{a}{2}} + \left(1+\frac{k_A}{k_B}\right)\frac{1}{1-\frac{k_A}{k_B}}\left[\dots\right] = (\dots)$$

$$\omega_- A_1 e^{-ik\frac{a}{2}} + \frac{\omega_+}{\omega_-}\left(-\omega_+ A_1 e^{-ik\frac{a}{2}} + \omega_+ C_1 e^{ik\frac{a}{2}} e^{-ik_B a}\right) = \omega_- C_1 \underbrace{}_{e^{ik\frac{a}{2}} e^{+ik_B \dots}}$$

$$\left(\omega_- - \frac{\omega_+^2}{\omega_-}\right)A\, e^{-ik\frac{a}{2}} = C e^{ik\frac{a}{2}}\left(\omega_- e^{+ik_B a} - \frac{\omega_+^2}{\omega_-} e^{-ik_B a}\right)$$

$$\left(1-\left(\frac{\omega_+}{\omega_-}\right)^2\right)A\, e^{-ik\frac{a}{2}} = C e^{ik\frac{a}{2}}\left(e^{ik_B a} - \left(\frac{\omega_+}{\omega_-}\right)^2 e^{-ik_B a}\right)$$

$$\frac{\omega_+}{\omega_-} = \frac{1+\frac{k_A}{k_B}}{1-\frac{k_A}{k_B}} = \frac{k_B+k_A}{k_B-k_A} = \frac{k_B+k}{k_B-k}$$

$$1-\left[\frac{\omega_+}{\omega_-}\right]^2 = \left(1+\frac{\omega_+}{\omega_-}\right)\left(1-\frac{\omega_+}{\omega_-}\right) =$$

$$= \frac{k_B - \cancel{k} + k_B + \cancel{k}}{k_B - k}\cdot\frac{\cancel{k_B} - k - \cancel{k_B} - k}{k_B - k} =$$

$|\dots|^2$

$$= \frac{-4k_B k}{(k_B-k)^2} = \frac{4k_B k}{(k-k_B)^2}$$

$k, k_B \in \mathbb{R}$

$$\left(\frac{4k_B k}{(k-k_B)^2}\right)^2 |A|^2 = |C|^2 \left\{\left[e^{ik_B a} - \left(\frac{k_B+k}{k_B-k}\right)^2 e^{-ik_B a}\right] \times\right.$$

$$\times\left[\dots \left(\frac{k_B+k}{\dots}\right)^2 e^{+ik_B a}\right]^2$$

$(1-w^2)^2$

$w = \frac{w_+}{w_-}$

$\times \left[e^{-ik_Ba} - \left(\frac{\ldots}{k_B-k}\right) e^{\ldots} \right] \}$

$$= 1 - \underbrace{\left(\frac{k_B+k}{k_B-k}\right)^2}_{w^2} \underbrace{\left(e^{-2ik_Ba} + e^{+2ik_Ba}\right)}_{\substack{2\cos(2k_Ba) \\ = 2(1-2\sin^2(k_Ba))}} + \underbrace{\left(\frac{k_B+k}{k_B-k}\right)^4}_{w^4} =$$

$$(1-w^2)^2|A|^2 = |C|^2 \{ \underbrace{1 - w^2\, 2\left(1-2\sin^2(k_Ba)\right) + w^4}_{\substack{1 - 2w^2 + 4w^2\sin^2(k_Ba) + w^4 = \\ = (1-w^2)^2 + 4w^2\sin^2(k_Ba)}} \}$$

$$\frac{|C|^2}{|A|^2} = \frac{(1-w^2)^2}{(1-w^2)^2 + 4w^2\sin^2(k_Ba)}$$

$$\frac{|C|^2}{|A|^2} = \frac{1}{1 + \frac{4w^2}{(1-w^2)^2}\sin^2(k_Ba)}$$

$$\frac{4w^2}{(1-w^2)^2} = \frac{\frac{4(k_B+k)^2}{(k_B-k)^2}}{\frac{(4k_Bk)^2}{(k-k_B)^4}} = \frac{(k_B+k)^2(k-k_B)^2}{4\,k_B^2k^2} = \frac{(k^2-k_B^2)^2}{4k_B^2k^2} =$$

$$= \frac{1}{4}\left(\frac{k^2-k_B^2}{k\,k_B}\right)^2 = \underline{\frac{1}{4}\left(\frac{k}{k_B} - \frac{k_B}{k}\right)^2}$$

$$\Rightarrow \boxed{\left|\frac{C}{A}\right|^2 = \frac{1}{1 + \frac{1}{4}\left(\frac{k}{k_B} - \frac{k_B}{k}\right)\sin^2(k_Ba)}}$$

→ pro velmi nízkou bariéru: $V_0 \to 0$

$$k_B = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \longrightarrow \sqrt{\frac{2mE}{\hbar^2}} = k_A = k_C = k$$

dostaneme $T = 1$

(B) částice má $E < V_0$

→ KM: odrazí se

→ QM: nenulová pst. průchodu bariérou

$$\psi_A(x) = A_1 e^{ik_A x} + A_2 e^{-ik_A x} \qquad x \le -\frac{a}{2}$$

$$\psi_B(x) = B_1 e^{ik_B x} + B_2 e^{-ik_B x} \qquad -\frac{a}{2} \le x \le \frac{a}{2}$$

$$\psi_C(x) = C_1 e^{ik_C x} + C_2 e^{-ik_C x} \qquad \frac{a}{2} \le x$$

$$k_A = k_C = \sqrt{\frac{2mE}{\hbar^2}}$$

$$k_B = \sqrt{\frac{2m(E-V_0)}{\hbar^2}} \xRightarrow{E < V_0} \varrho_B = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

$$k_B = i\varrho_B \qquad \varrho_B = -ik_B$$

částice přilétá z $-\infty \Rightarrow C_2 = 0$

sešívací podmínky:

- $\psi_A(-\frac{a}{2}) = \psi_B(-\frac{a}{2})$ a $\psi'_A(-\frac{a}{2}) = \psi'_B(-\frac{a}{2})$
- $\psi_B(\frac{a}{2}) = \psi_C(\frac{a}{2})$ a $\psi'_B(\frac{a}{2}) = \psi'_C(\frac{a}{2})$

$\Rightarrow$ dostaneme stejné rce jako výše
(ale $k_B \to i\rho_B$)

⋮

$\Rightarrow$ dostaneme $k_B \to i\rho_B$

$\omega \in \mathbb{C}$

$$\underbrace{\left|1-\left(\frac{i\rho+k}{i\rho-k}\right)^2\right|^2}_{\omega^2}|A|^2 = |C|^2 \underbrace{\left\{\left[e^{-\rho a} - \left(\frac{i\rho+k}{i\rho-k}\right)^2 e^{+\rho a}\right] \times \left[e^{-\rho a} - \left(\frac{-i\rho+k}{-i\rho-k}\right)^2 e^{+\rho a}\right]\right\}}$$

ω, ω^*

$$|1-\omega^2|^2 = (1-\omega^2)(1-(\omega^*)^2)$$
$$= 1 - (\omega^2 + (\omega^*)^2) + 1$$
$$= 2 - (\omega^2 + (\omega^*)^2)$$
$$= e^{-2\rho a} - (\omega^2 + (\omega^*)^2) + \underbrace{|\omega|^4}_{\Rightarrow = 1} e^{2\rho a}$$

$$|\omega|^2 = \frac{+i\rho+k}{i\rho-k} \cdot \frac{-i\rho+k}{-i\rho-k} = \frac{\rho^2+k^2}{k^2+\rho^2} = 1$$

$\cosh(x) = \frac{1}{2}(e^x + e^{-x})$

$$= \underbrace{2\cosh(2\rho a)}_{2(1+2\sinh(\rho a)} - \underbrace{(\omega^2 + (\omega^*)^2)}_{}$$

$$\left(\frac{i\rho+k}{i\rho-k}\right)^2 = \frac{k^2+\rho^2+2ik\rho}{k^2+\rho^2-2ik\rho}$$

$$\left(\frac{-i\rho+k}{-i\rho-k}\right)^2 = \frac{k^2+\rho^2-2ik\rho}{k^2+\rho^2+2ik\rho}$$

$$\frac{|C|^2}{|A|^2} = \frac{2-(w^2+(w^*)^2)}{2+4\sinh(\rho a)-(w^2+(w^*)^2)} =$$

$$= \frac{1}{1+\frac{4}{2-(w^2+(w^*)^2)}\sinh(\rho a)}$$

$$\Rightarrow \frac{1}{4}\left(\frac{k_A}{\rho_B}+\frac{\rho_B}{k_A}\right)^2$$

$$w^2 = \left(\frac{i\rho+k}{i\rho-k}\right)^2 = \frac{(i\rho+k)^4}{k^2+\rho^2}$$

$$(w^*)^2 = \left(\frac{-i\rho+k}{-i\rho-k}\right)^2 = \frac{(-i\rho+k)^4}{k^2+\rho^2}$$

$$\Rightarrow \frac{(i\rho+k)^4+(-i\rho+k)^4}{k^2+\rho^2} = \frac{\left((i\rho+k)^2+(-i\rho+k)^2 i\right)\left((i\rho+k)^2-i(-i\rho+k)^2\right)}{k^2+\rho^2}$$

$$= \frac{\left(\rho^2+k^2+2i\rho k+(\rho^2+k^2-2i\rho k)i\right)\times\left(\rho^2+2i\rho k+k^2-i(\rho^2+k^2-2i\rho k)\right)}{k^2+\rho^2} =$$

$= \dots$

$$\boxed{\left|\frac{C}{A}\right|^2 = \frac{1}{1+\frac{1}{4}\left(\frac{k}{\rho}+\frac{\rho}{k}\right)^2\sinh(\rho a)}}$$

$\Rightarrow$ nenulová pst. průchodu bariérou

$\Rightarrow$ širší bariéra ($a\uparrow$) $\Rightarrow$ menší pst. průchodu