ÚDKM cv. 8 DÚ

(1) $\hat{H} = \underbrace{\frac{1}{2}(\hat{x}^2 + \hat{p}^2)}_{H_0} + \delta \underbrace{\hat{x}^2}_{V}$

- vyjádříme poruchu $\hat{x}^2$ pomocí an./kr. op. $\hat{a}/\hat{a}^+$:

$$\hat{x} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{a}^+)$$

$$\hat{x}^2 = \frac{1}{2}(\hat{a} + \hat{a}^+)(\hat{a} + \hat{a}^+) =$$

$$= \frac{1}{2}(\hat{a}\hat{a} + \hat{a}^+\hat{a} + \hat{a}\hat{a}^+ + \hat{a}^+\hat{a}^+)$$

- pro působení $\hat{x}^2$ na n-tý stav LHO:

$$\hat{x}^2|n\rangle = \frac{1}{2}(aa + a^+a + aa^+ + a^+a^+)|n\rangle$$

$$= \frac{1}{2}\left(\underline{\sqrt{(n-1)n}\,|n-2\rangle} + \underline{(2n+1)|n\rangle} + \sqrt{(n+1)(n+2)}\,|n+2\rangle\right)$$

- první oprava pro zákl. stav $E_0^{(0)} = \frac{1}{2}$

$$\underline{\underline{E_0^{(1)}}} = \langle 0|\hat{V}|0\rangle = \langle 0|\hat{x}^2|0\rangle = \underline{\underline{\frac{1}{2}}}$$

- druhá oprava pro zákl. stav:

$$\underline{\underline{E_0^{(2)}}} = \sum_{k\neq 0} \frac{|\langle n|x^2|0\rangle|^2}{\underset{\frac{1}{2}}{E_0^{(0)}} - \underset{k+\frac{1}{2}}{E_k^{(0)}}} = \sum_{k\neq 0} \frac{|\langle n|x^2|0\rangle|^2}{-k} =$$

přispívá pouze $k=2$

$$= \frac{\left|\frac{1}{2}\sqrt{2}\right|^2}{-2} = \underline{\underline{-\frac{1}{4}}}$$

$\Rightarrow$ energie do druhého řádu

$$\underline{\underline{E_0}} \approx E_0^{(0)} + \delta E_0^{(1)} + \delta^2 E_0^{(2)}$$
$$= \underline{\underline{\frac{1}{2} + \delta \frac{1}{2} + \delta^2 \left(-\frac{1}{4}\right)}}$$

② $$H = \underbrace{\frac{1}{2}(p_x^2 + x^2) + \frac{1}{2}(p_y^2 + y^2)}_{= H_0 = H_{0x} + H_{0y}} + \delta xy$$

$$H_0 |n_x, n_y\rangle = (H_{0x} + H_{0y}) |n_x n_y\rangle =$$
$$= (H_{0x} |n_x\rangle) |n_y\rangle + |n_x\rangle (H_{0y} |y\rangle) =$$
$$= \left(n_x + \frac{1}{2} + n_y + \frac{1}{2}\right) |n_x n_y\rangle =$$
$$= (n_x + n_y + 1) |n_x n_y\rangle$$

$$\underline{\underline{E_{n_x n_y}^{(0)} = n_x + n_y + 1}}$$

první excitovaný stav: $|10\rangle, |01\rangle$ $\quad \underline{E_1^{(0)} = 2}$

$$\langle n_x' n_y' | xy | n_x n_y\rangle = \langle n_x' | x | n_x\rangle \langle n_y' | y | n_y\rangle =$$

$$x = \frac{1}{\sqrt{2}}(a_x + a_x^\dagger) \qquad a|n\rangle = \sqrt{n}\,|n-1\rangle$$
$$y = \frac{1}{\sqrt{2}}(a_y + a_y^\dagger) \qquad a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

$$= \frac{1}{2} \langle n_x' | a_x + a_x^\dagger | n_x\rangle \langle n_y' | a_y + a_y^\dagger | n_y\rangle =$$

$$= \frac{1}{2}\left(\sqrt{n_x}\langle n_x'|n_x-1\rangle + \sqrt{n_x+1}\,\langle n_x'|n_x+1\rangle\right)\times$$

$$\times\left(\sqrt{n_y}\langle n_y'|n_y-1\rangle + \sqrt{n_y+1}\,\langle n_y'|n_y+1\rangle\right) =$$

$$\begin{pmatrix} \langle 10|V|10\rangle & \langle 10|V|01\rangle \\ \langle 01|V|10\rangle & \langle 01|V|01\rangle \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$

$$\Rightarrow \det\begin{vmatrix} -E_1^{(1)} & 1/2 \\ 1/2 & -E_1^{(1)} \end{vmatrix} = \left(E_1^{(1)}\right)^2 - \frac{1}{4} \stackrel{!}{=} 0$$

$$\underline{\underline{E_1^{(1)} = \pm\frac{1}{2}}}$$

$$E_1 \approx E_1^{(0)} + \delta E_1^{(1)} = 2 \pm \frac{1}{2}\delta$$