

$$\begin{aligned}
 \textcircled{\text{I}} \quad \underline{H_0 |ij\rangle} &= (H_{0,x} |i\rangle_x) |j\rangle_y + |i\rangle_x (H_{0,y} |j\rangle_y) = \\
 &= (i + \frac{1}{2}) |i\rangle_x |j\rangle_y + (j + \frac{1}{2}) |i\rangle_x |j\rangle_y = \\
 &= (i + j + 1) |i\rangle_x |j\rangle_y = \underline{\underline{(i+j+1) |ij\rangle}}
 \end{aligned}$$

$$\textcircled{\text{II}} \quad E_1 = 2 \Rightarrow |ij\rangle = \{|10\rangle, |01\rangle\}$$

$$\begin{pmatrix} W_{11} - E_1^{(0)} & W_{12} \\ W_{21} & W_{22} - E_1^{(0)} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = 0$$

$$\underline{W_{11}} = \langle 10 | V | 10 \rangle = \langle 10 | x^2 y^2 | 10 \rangle =$$

$$= \langle 1 | x^2 | 1 \rangle_x \langle 0 | y^2 | 0 \rangle_y =$$

$$= \frac{1}{2} (\langle 1 | x a_x a_x^\dagger | 1 \rangle + \langle 1 | x a_x^\dagger a_x | 1 \rangle) \times$$

$$\times \frac{1}{2} (\langle 0 | y a_y a_y^\dagger | 0 \rangle) =$$

$$= \frac{1}{4} (2+1) \cdot 1 \quad (\text{ostatní jsou } 0)$$

$$= \underline{\underline{\frac{3}{4}}}$$

$$\text{z symetrie } W_{22} = \langle 01 | x^2 y^2 | 01 \rangle = \langle 10 | x^2 y^2 | 10 \rangle = \underline{\underline{\frac{3}{4}}}$$

$$\underline{W_{12}} = \langle 10 | x^2 y^2 | 01 \rangle = \langle 1 | x^2 | 0 \rangle \langle 0 | y^2 | 1 \rangle = 0 = W_{21}$$

$\Rightarrow$ dostaneme matici:

$$\begin{pmatrix} \frac{3}{4} - E_1^{(0)} & 0 \\ 0 & \frac{3}{4} - E_1^{(0)} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} = 0$$

$$\textcircled{\text{III}} \quad E_3 = i+j+1 = 4:$$

4 stavů: $|30\rangle, |21\rangle, |12\rangle, |03\rangle$

$$\Rightarrow \text{matice } \begin{pmatrix} \langle 30 | V | 30 \rangle & \langle 30 | V | 21 \rangle & \langle 30 | V | 12 \rangle & \langle 30 | V | 03 \rangle \\ \langle 21 | V | 30 \rangle & & & \\ \langle 12 | V | 30 \rangle & & \text{add.} & \\ \langle 03 | V | 30 \rangle & & & \end{pmatrix}$$

dosažením za $\hat{x}^2 = \frac{1}{2} (\hat{a}^\dagger \hat{a}^\dagger + \hat{a} \hat{a}^\dagger + \hat{a}^\dagger \hat{a} + \hat{a} \hat{a})$ a působením $a^\dagger |n\rangle, a |n\rangle$ (viz výše)
dostaneme matici (+ použijeme symetrii $|30\rangle \sim |03\rangle, |12\rangle \sim |21\rangle$)

$$\begin{pmatrix} \frac{7}{4} & 0 & \frac{\sqrt{3}}{2} & 0 \\ 0 & \frac{15}{4} & 0 & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & 0 & \frac{15}{4} & 0 \\ 0 & \frac{\sqrt{3}}{2} & 0 & \frac{7}{4} \end{pmatrix}$$

$\Rightarrow$ ~~diagon~~ vlastní čísla $\frac{1}{4} (11 \pm 2\sqrt{7})$

$$\Rightarrow \text{energie } E \approx E_3^{(0)} + \delta E_3^{(1)} = 4 + \delta \frac{1}{4} (11 \pm 2\sqrt{7})$$

částečné
sejmuví
degenerace